1973

Factors of mathematical aptitude

Christine Leigh Taylor
Iowa State University

74-580

TAYLOR, Christine Leigh, 1944-
FACTORS OF MATHEMATICAL APTITUDE.

Iowa State University, Ph.D., 1973
Education, psychology

University Microfilms, A XEROX Company, Ann Arbor, Michigan

Factors of mathematical aptitude

by

Christine Leigh Taylor

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major: Education (Evaluation)

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Area

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1973

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Due to differential learning patterns, one method of instruction does not necessarily bring success to all students. In order to optimize learning, instruction needs to be adapted to each individual's patterns of abilities. To date, most research studies on instruction have manipulated several treatments and compared average group differences in achievement scores. Treatments or instructional programs producing the highest average gain are then implemented by the schools. Since new programs (for example, foreign language laboratories) do not report success for all students, the search for the one "best" method or instructional treatment continues (Salamon, 1972).

What has been overlooked in these studies is the influence of the unique set of aptitudes brought to the learning situation by the individual student and the interaction of these aptitudes with the instructional treatments. According to the theory of Aptitude-Treatment Interaction (ATI), "for every person there is a best treatment, and for every treatment a best type of person" (Cronbach & Gleser, 1965, p. 143). In other words, treatments should be selected on the basis of their ability to produce maximum success for specific types of individuals.

To achieve this goal, Glaser (1972, p. 8) stated that "what is required is a measure of aptitude that predicts who will learn better from one curriculum or method of learning than from another." Scholastic aptitude tests are not entirely satisfactory for this purpose since they "account for only 35 to 45 percent of the variation in school performance"

(Glaser, 1972, p. 7). In searching for measures of aptitude which will have higher predictive value, traditional concepts of aptitude will change (Glaser, 1972) and so must the existing aptitude measures (Cronbach, 1957). At present, "our generally used aptitude constructs are not productive dimensions for measuring those individual differences that interact with different ways of learning" (Glaser, 1972).

With respect to ATI, the term, "aptitude", includes all characteristics of individuals, rather than being limited to the common singular concept of aptitude. Aptitude then is "a complex of personal characteristics that accounts for an individual's end state after a particular educational treatment, i.e., that determines what he learns, how much he learns, or how rapidly he learns" (Cronbach, 1967, p. 23). Cronbach also hypothesized that aptitude "may have as much to do with styles of thought and personality variables as with the abilities covered in conventional tests" (Cronbach, 1967, p. 24).

The topic of concern in this study is mathematical aptitude. As viewed by Cronbach (1967, p. 27) "we haven't the faintest evidence, for example, what constitutes mathematical aptitude, save for the obvious fact that a person who has mastered one mathematical fact or process has an advantage in learning the next process in a hierarchy." The primary purpose of this research study is to define, more precisely, the factors of mathematical aptitude. However, since aptitude information becomes more useful when we know how it interacts with the given treatments (Cronbach, 1967), the secondary purpose is to study the interactions of the variables comprising mathematical aptitude with two treatments, algebra and geometry. The term, "algebra", refers to the first year course in algebra, which generally includes the use of the quadratic formula in factoring; "geometry" refers to the full year course in plane Euclidean geometry.

According to Bracht (1970, p. 639), "to be differentially effective for various types of students, the alternative treatments should demand different abilities for successful performance." The function and major differences of these instructional treatments have been described by Salamon (1972, p. 340) in his preferential model for aptitude-treatment interactions. Treatment in his model "call upon and utilize learner's higher aptitudes, neither making up for deficiencies nor compensating for them. Differences may be in content, structure, modality of presentation, etc." Success can be predicted from this model "when an aptitude in which he [a student] is proficient is called upon" (Salamon, 1972, p. 340). For example, if having good spatial perception enhances learning in geometry, a student's success in geometry could be predicted from his spatial perception ability scores. If the student were found to be deficient in this ability, alternative treatments, which are either compensatory or remedial, could be given to the student in order to maximize his learning and success.

The problem in this study is to identify the principal variables which are, theoretically and/or empirically, related to mathematical aptitude and which interact with the two treatments, algebra and geometry. These two courses are two distinct branches of study in mathematics. The independence of these courses is exemplified by the student's mathematics curriculum. A student may select either course as the point of entry into higher mathematics and it is possible to omit one of these courses and not experience failure in subsequent courses. It appears that algebra and geometry require different abilities for successful performance within each of the two courses.

The variables to be considered in this study are: problem solving ability, subject matter interests (mathematics, reading, algebra, geometry); differential abilities (verbal reasoning, numerical ability, abstract reasoning, space relations); personality factors (planning and organizing, intellectually oriented, persistence, self-confidence, conformity, having cultural interests, seeking new experiences, liking to be alone, being a perfectionist); general aptitude test scores (verbal, mathematical); musical experience (vocal, instrumental); IQ; sex; socioeconomic status (parents' educational and occupational levels); and success in mathematics (grade point average in algebra and geometry, class rank, teacher ratings, standardized test scores).

It is hypothesized that some of these variables will be more critical for success in mathematics than others. In addition, it is hypothesized that certain variables will be more characteristic of successful geometry students, while other variables will better describe successful algebra students, i.e., there will be an aptitude-treatment interaction.

The following operational definitions will be used in this study: 1) math grade point average (MGPA) is the average of all semester grades in algebra and geometry; 2) "star" students are those students who have demonstrated high mathematical ability (MGPA: 3.00 to 4.00) and who are rated high in math aptitude (5.5 to 7.0) by their former math teachers; 3) "nonstar" students are those students who have shown low in ability in math (MGPA: 0.00 to 1.75) and who are rated low in math aptitude (1.0 to 3.5) by their former math teachers; 4) "algebraic stars" are those students whose algebra GPA was greater than their geometry GPA; and 5) "geometric stars" are those students whose geometry GPA was greater than their algebra GPA.

REVIEW OF LITERATURE

Theoretical Background

The concept of individual differences has appeared in the literature from the time of Plato to the present (Anastasi, 1958). However, it was not until the development of intelligence tests in the early 1900's that differences in students' mental abilities could be measured. Still, the inability of educators to properly place students in curricula where each could experience success was evidenced by the failure of many students. Overemphasis on this golden ratio, IQ, signaled the need for other measures of mental ability which could aid the student to select courses where he could attain success. "The research and theories of Thorndike, Kelley, Spearman, Thomson, Thurstone, and others have made us increasingly aware that so-called intelligence is not a unitary trait--it is composed of many abilities, which are present in different individuals in varying amounts" (Bennett, Seashore, & Wesman, 1952, p. 1).

Application of the data from differential aptitude tests to the educational process has assisted educators in predicting success (or failure) for each individual. "The aim of prediction studies is to estimate, in advance of participation, the level of an individual's performance in a given activity" (Horst, 1941, p. 12). To predict successful performance requires the determination of the personal and situational factors related to success in the specific activity. The prediction process, as described by Horst (1941), contains the following steps: 1) determination of the criterion for success in an activity;

2) isolation and analysis of those personal or situational factors associated with individual differences prior to the activity; and 3) prediction for success or failure based on the preceding analysis.

Counseling eighth grade students on their decisions to take Algebra I can be used as an example of this process. The criterion for success in Algebra might be defined as obtaining a grade of "C" or higher. Counselors may then make the observation that very few students have succeeded in Algebra I if they had a low IQ and/or received low grades in Math 8. Students could then be advised to take General Math 9, rather than Algebra I, if their Math 8 grades and/or IQ scores fell below a critical level.

This system is not very adaptable to the student as different methods of learning for students of varying background and abilities are limited. With this "selective" mode, "the fixed or limited paths available require particular student abilities, and these particular abilities are emphasized and fostered to the exclusion of other abilities" (Glaser, 1972, p. 2).

"In contrast to a selective mode, an 'adaptive' mode of education assumes that the educational environment can provide for a wide range and variety of instructional methods and opportunities for success. Alternative means of learning are adaptive to and are in some way matched to knowledge about each individual--his background, talents, interests, and the nature of his past performance" (Glaser, 1972, p. 2). Thus, in order to adapt instruction to an individual, it is necessary to know the individual's special aptitudes and the type of treatment or situational factors which will maximize the chances of his success. This process of adjusting the instruction to fit the unique characteristics of each student is the essence of Aptitude-Treatment Interaction (ATI) theory (Cronbach, 1957).

To identify those treatments which tend to increase student success for specific aptitudes, a process analysis should be conducted (Cronbach & Snow, 1969). This analysis involves the following steps: 1) observation of the learner performing the task; 2) identification of processes (treatment A) used; 3) estimation of the abilities required (ability A)[1] to perform the task; and 4) development of alternative treatments (B), which require different processes to be used to attain the same goals. The ability (B) to perform the second set of processes (treatment B) should be unrelated, or only moderately related, to the ability A needed to perform treatment A. The interaction between aptitudes and treatments is then tested. This interaction is said to be disordinal if the differences between alternative treatments at two levels of a personological variable are both significantly nonzero and different in 'algebra sign' (Bracht, 1969).

ATI Studies

Thus far, very few studies designed to test ATI theory have been successful in finding significant disordinal interactions (Aiken, 1971; Bracht, 1970).

---

[1] The term, ability, may be considered plural and to cover a variety of personality characteristics, aptitudes, interests, etc.

Bracht (1969) surveyed 90 studies in an attempt to define the reasons for the paucity of disordinal interactions. Each study was classified according to the type of controls used in the experiment, degree of complexity of the personological variables, and the specificity of the dependent variables. Only five studies showed the required interactions. Bracht's criticism of the 85 studies having no disordinal interations centered on their approach to the selection of the personological variables. He believes that most investigators have tried to find ATIs by trial and error method. In order to obtain disordinal interactions, he suggests using process analysis and two or more personological variables. The intercorrelation of these variables should be low, with one variable correlating highly with the first treatment and the other variable correlating highly with the second treatment. He suggests that "the real test for the concept of ATI will come as more experimenters use process analysis for developing alternative treatments" (1970, p. 640).

One relevant study having disordinal interactions was conducted by Bracht (1969). Sixth grade students were divided into groups and were taught addition of integers by means of self-instructional materials; one set of materials emphasized spatial ability and the other verbal ability. Although the results were not consistent, there was some evidence that the verbal treatment was superior for students with low spatial ability and the spatial treatment was superior for students with low verbal ability.

Other studies involving learning of mathematics which have attempted to find disordinal interactions are described below.

In a study by Becker (1970), 70 Algebra I students were taught the summing of a number series by two different methods. Students were matched according to verbal and math ability test scores and were randomly assigned to treatments. Using programmed materials, half of the sample were presented the correct formula for an associated number series and a verbal explanation; the other half were taught the process of developing the formula by working the series using small steps. Becker found no significant interactions between aptitudes and treatments.

Carey (1958) studied 191 geometry students looking for possible interactions of two types of programmed materials with general reasoning (Necessary Arithmetic Operations) and visualization (Paper Form Board) abilities. Quadratic inequalities were presented by a graphical method and by an analytical method (i.e., properties of signed numbers). The instruction lasted for two class periods. It was found that both treatments were equally effective for learning as evidenced by students' total scores on a transfer of training test. A few of the test items seemed to indicate possible treatment interactions; however, the experimental tests were not pretested, thus making the results questionable since the achievement test was found to have low reliability.

Davis (1966) investigated the learning of programmed materials on derivatives of algebraic expressions and vector multiplication by semantic and symbolic approaches. High school tenth graders were included with a group of undergraduate students. This study showed interactions between ability and content format. Maximum achievement occurred when the content format was similar to the pattern of abilities, as measured by Guilford's structure-of-the-intellect battery. Both the semantic and symbolic factor tests significantly predicted achievement on the symbolic learning materials posttest. Davis concluded that materials which matched a student's aptitudes produced better learning.

Variables Related to Math Achievement

Many studies have been concerned with the prediction of success in mathematics (see Table 1). Math achievement has usually been defined by course grades and/or standardized achievement test scores. A brief description of some of these studies is presented in the following section.

Intelligence and differential abilities

To be successful in mathematics Wrigley (1958) stated that the first requirement was high general intelligence. Muscio (1962) has added high verbal ability to this requirement.

Suydam's (1970) survey of research on secondary school mathematics showed 31 studies of achievement in mathematics as related to general intelligence. She found that the correlation between IQ and arithmetic test scores varied from 0.30 to 0.67 across studies. This inconsistency in the magnitude of the correlation coefficient is due to variations in instruments, samples, and procedures. Correlations above 0.70 have been observed between intelligence test socres and math grade placement.

In other studies, Hummer (1936) found a correlation of 0.58 between geometry test scores and IQ; arithmetic achievement test scores have

Table 1. Summary of studies with variables related to math achievement

Variables in study	Sample size	Grade level	Author(s) (year)
Intelligence and Differential Abilities			
CTMM (IQ) and Cooperative Elementary Algebra Test	635	9	Rosilda (1951)
Otis Group Intelligence Scale, Columbia Research Bureau Plane Geometry Test	153	10	Hummer (1936)
Kuhlmann-Anderson Battery, California Achievement Battery	293	8-12	Jacobs (1957)
Algebra grades, DAT	517	9-12	Cain (1966)
Personality			
Sixteen Personality Factor Questionnaire, Math grade point average	75	10	Ayers, Bashaw, & Wash (1969)
Culture Fair Intelligence Test, High School Personality Questionnaire, ETS Standardized Achievement Tests	311	6,7	Barton, Dielman, & Cattell (1972)
CTMM, Stress Questionnaire, Cowan & Mandler's Test Anxiety Scale	132	8,9	Callister (1965)
High School Personality Questionnaire, math achievement measures	--	6,7	Cattell, Butcher (1968)
Taylor Manifest Anxiety Scale, Dutton's Scale of Attitudes toward Arithmetic, student ranking of school subjects	44	8	Degnan (1967)

Table 1 (Continued)

Variables in study	Sample size	Grade level	Author(s) (year)
Woody-McCall Test in Mixed Fundamentals and Questionnaire, IQ	640	9	Mallory (1939)
Math Attitude Scale, Semantic Differential on Math Attitudes	68	18	McCallon, Brown (1971)
IQ, student attitudes toward arithmetic, grades, teacher attitudes toward arithmetic	306 Ss, 59 Ts	7 Ss in-service, Ts	Phillips (1969)
IQ, grades, socioeconomic status, Self Concept of Ability Scale, Occupational Aspiration Scale	201 males	9	Schneider (1969)
ETS Personality Research Inventory, ETS SAT-Math	1987	12	French (1962)
Rorschach, Children's Apperception Test, Thematic Apperception Test, Draw a Person Test, Sentence Completion Test, Primary Mental Abilities Tests, Stanford-Binet, Wechsler-Bellevue, teacher-made achievement tests	45 (gifted)	7	Haggard (1957)
Grade point averages, personality measures	434	9-11	Kochnower (1961)
Test Anxiety Questionnaire, Otis Quick-Scoring Mental Ability Test, Introduction to Vector Geometry Test	6 classes	10	Morgan (1965)
<u>Interest in Mathematics</u>			
Mathematics Questionnaire	531	8,13,17	Aiken (1972a)

Table 1 (Continued)

Variables in study	Sample size	Grade level	Author(s) (year)
MAS (Math Attitude Scale), DAT Verbal, DAT Numerical Ability	127	13	Aiken, Dreger (1961)
ITBS-Arithmetic, ITED-Quantitative Thinking, Questionnaire of math attitudes, math grade point average	607	11,12	Anttonen (1967)
IQ, GPA, ITED, Mathematics Attitude Inventory, teacher ratings on student attitudes	755	7-12	Ellingson (1962)
Sex, math interest and grades, math electives	4 schools	10,11	Farley (1969)
Pittsburgh Temperment Inventory, Cooperative tests, Otis IQ test, programmed unit tests, student attitude measure	616	7,9	Ferderbar (1965)
Math tests, interest inventory	425 schools	8	Husén (1967)
Age, race, socioeconomic status, interest inventory	2234	4-8	Kauffman (1955)
MAS, Brown's CAI Attitude Questionnaire, Thomas' CAI Attitude Questionnaire, math achievement test	243	13-16	Kochler (1972)
<u>Reading Ability</u>			
Otis Gamma, Cooperative Reading Test, DAT	44	11	Call, Wiggin (1966)
IQ, algebra prognosis test, reading comprehension test	(not reported)	9	Clark (1939)

Table 1 (Continued)

Variables in study	Sample size	Grade level	Author(s) (year)
IQ, Traxler Silent Reading Test, Traxler Arithmetic Test	132	7	Curry (1955)
Pressy et al. arithmetic and algebra vocabulary tests, geometry test, investigator's math test	500	7-9	Dresher (1934)
Otis Test of Mental Ability, Gates Reading Survey, Stanford Achievement Test, math vocabulary test, test on graphs and equations	319	9	Eagle (1948)
Otis Self-Administering Tests of Mental Ability, Stanford Achievement Test	42	1-8	Finck (1935)
Orleans Geometry Prognosis Test, Metropolitan Achievement Test-Adv. Reading Form, ITED-Quantitative Thinking, NY Regent's Examination on Tenth Year Mathematics	158	9,10	Posamentier (1966)
<u>Problem Solving</u>			
Luchins' water jar problems, jigsaw puzzles	48	6	Ackerman, Levin (1958)
Stanford Arithmetic Reasoning and Reading tests, Otis Self-Adm. Test of MA, investigator's 5-item problem situations test	564	7-9	Bowman (1932)
Word problems, aptitude, achievement tests, and attitude scales	56	8	Kilpatrick (1967)

Table 1 (Continued)

Variables in study	Sample size	Grade level	Author(s) (year)
ITBS, Lorge-Thorndike Intelligence Test	1107	4,8	Martin (1963)
IQ, grades, verbal ability, math test	211	7	Werdelin (1966)
Music			
Drake Music Aptitude Test, DAT Numerical Ability Test, CTMM, Stanford Scientific Aptitude	256	high school	Jenkins (1961)
Socioeconomic Status			
Survey of several studies	--	--	Anastasi (1958, 1966)
SCAT, National Merit Scholarship Qualifying Test, grades, AVLSV, teacher ratings on student personality	76 (gifted)	12	Johnson (1966)
Sex			
SRA Arithmetic Reasoning Test, IQ	623 (320 male)	7	Alexander (1962)
Terman Group Test of Mental Ability, Otis Self-Adm. Test of MA, math grades	191	9	Burgert (1935)
Webb Geometry Test, Terman Group Test of MA	873 (436 male)	10	Foran, O'Hara (1935)

Table 1 (Continued)

Variables in study	Sample size	Grade level	Author(s) (year)
Pre-Instruction achievement test, Dinkel's Survey of Algebraic Aptitude Test, IQ, SRA-Reading Test	107 (57 male)	9	Sheehan (1968)
Iowa Every-Pupil Test	2450	3-8	Stroud, Lindquist (1942)

been found to correlate higher with IQ than did reading achievement scores (Jacobs, 1957); and Rosilda (1951) obtained a correlation of 0.42 between IQ and algebra test scores. In general, it can be said that achievement in mathematics is positively correlated with intelligence.

Achievement in math may be due to differential abilities. Cain (1966) found the relationship of Algebra I grades to the Numerical test score of the DAT to be significantly higher than the relationship with Verbal test score.

Personality

"Since only about 50% of the score variance on measures of mathematical achievement can be accounted for by tests of general and special abilities it seems reasonable to expect that scores on personality tests and inventories would improve the predictability of mathematics achievement" (Aiken, 1973, p. 20).

Several personality factors have been found to be important in predicting mathematics achievement. For example, adventurousness, as measured by the High School Personality Questionnaire, was significantly related to math achievement for sixth and seventh graders (Cattell & Butcher, 1968). They also found that the emotional stability of the seventh graders correlated highly with reading achievement as well as achievement in mathematics.

In another study (Barton, Dielman & Cattell, 1972), IQ, as measured by the Culture Fair Intelligence Test, accounted for 20-30% of the variation in math achievement scores. When personality measures were included in the analysis, the amount of explained variation doubled.

Anxiety, as a dimension of personality, has been measured in several studies. Degnan (1967) found underachievers to have less positive interest in math and lower anxiety level than achievers. Girls who felt anxious tended to do well on math tests (French, 1962). Callister (1965) compared programmed and traditional Algebra and Geometry courses and found anxiety to be unrelated to the method of instruction. Morgan (1965) compared programed instruction to teacher-taught classes and Algebra-Trig students' anxiety levels were examined after being subjected to either a "relaxed" or "high anxiety producing" environment (Zamboni, 1968); no significant differences were found in either study between anxiety and method of instruction.

Kochnower, as cited by Suydam (1972), found students with math averages greater than their overall grade averages were found to be more emotional and to be characterized as nonconformists.

In a study of 75 tenth grade students, Ayers, Bashaw, and Wash

(1969) found personality traits to correlate very low with math achievement. The better math students tended to be more withdrawn, conscientious, emotional, immature, and lacking in frustration tolerance.

Haggard (1957) found 7-year old gifted, high math achievers to be able to express their thoughts freely, to have feelings of being capable, to be more curious, and to have independence of thought.

The research results in this area have been summarized by Aiken who stated: "Unfortunately, due in no small degree to the limitations of psychological measuring instruments, affective measures have not usually contributed substantially to the predictive efficiency of cognitive variables. Nevertheless, it is recognized that personality traits do affect success in mathematics" (Aiken, 1973, p. 20).

Interest in mathematics

Shapiro (1961) characterized students' failure in math as a lack of interest or general dislike for mathematics, a lack of attention or study, lack of good work habits, and a lack of ability.

Attitudes toward mathematics have been measured in a variety of studies with varying aged students and utilizing different instruments. The results of these studies indicate that achievement and interest in mathematics are positively correlated (Ellingson, 1962; Schneider, 1969; Aiken, 1972a; McCallon & Brown, 1971; Aiken & Dreger, 1961).

Aiken (1972a) correlated biographical data obtained from 182 eighth grade students with a math attitude scale and found math attitudes to correlate with routine computations, terms, symbols and word problems; to differ from male to female students; to correlate positively with grades in arithmetic and junior high mathematics; and to relate to student perception of parent and teacher attitudes toward mathematics.

Attitudes toward math have been found to be more stable for high school students than for elementary school students (Anttonen, 1967; Aiken, 1972b).

Several studies have been concerned with the relationship of attitudes toward various instructional materials or methods. Ferderbar (1965) found a decrease in attitudes toward programmed instruction and toward mathematics in a study involving 616 junior high school students. Undergraduate students who learned math principles by Computed-Assisted Instruction (CAI), as contrasted with the traditional teacher approach, tended to show an increased interest in computers and a change toward a more favorable attitude in math (Kochler, 1972).

Math teachers have also been found to have an influence on students' interests in math. Phillips (1969) compared seventh graders' attitudes toward math to the attitudes of their present and former math teachers. The attitudes of the most recent teacher correlated significantly with student attitudes.

In surveying various curriculum adjustments in algebra designed for the slow learner, Mallory (1939, p. 150) concluded that "in planning a course for slow pupils this element of interest must receive adequate attention. Pupils will succeed better with work which they enjoy."

A positive correlation of attitudes toward math and math achievement has also been found in 12 countries in the international study as reported by Husén (1967).

Reading

"Research indicates that many problem-solving difficulties are actually reading difficulties" (PREP, 1972, p. 15). Eagle (1948) attempted to define those reading abilities which predict success in math. Those factors were: math vocabulary, ability to organize information, and interpretation of graphs and formulas. Posamentier (1966) found that students who read well scored better on a geometry test than students who read poorly.

Most of the other studies involving math and reading contrasted groups of students receiving special reading instruction with students having no instruction. Those students who were given reading instruction achieved more success than their peers in Algebra I (Clark, 1939), Algebra II (Call & Wiggin, 1966), Math 7-9 (Dresher, 1934), Arithmetic 1-8 (Finck, 1935). However, Curry (1955) conducted a similar experiment with Math 7 students and found no significant differences between groups of students.

Problem solving

Problem solving ability, as defined by Joan Suppes (1971, p. 1) is the ability "to recognize problems, generate hypotheses, reason out the implications of these hypotheses, and test them against experience." Although many studies contain the words, "problem solving" in their titles, most of these studies center around computational problems and do not fit the above definition. Thus, this review is concerned with those studies which incorporate verbal or situational problems.

In a study conducted by Kilpatrick (1967), eighth grade students were asked to solve word problems using equations. Their scores were found to correlate positively with math achievement, general reasoning, word fluency, and quantitative ability.

Two factor-analytic studies on problem solving yielded conflicting results; this may be due to the differences in the factor analysis procedure used, as Aiken (1972b) has suggested, and/or due to the differences in the problem solving instruments. Werdelin (1966) found problem solving tests loaded strongly on a general reasoning factor and to a lesser extent on numerical and deductive reasoning factors; spatial and verbal comprehension factors were found to be unrelated to problem solving. Martin (1963), however, found high correlations between problem solving and reading comprehension, computation, abstract verbal reasoning and arithmetic concepts.

Computer "programmers have found that the incorporation of general heuristic rules, such as working backward or using a diagram, not only facilitates problem solving, but also results in performance by the computer that closely resembles the behavior of humans struggling with similar problems" (Kilpatrick, 1969, p. 527).

When given the opportunity to solve problems by several different methods, sixth grade students were found to be more persistent and more flexible in the methods used than students who knew only one method (Ackerman & Levin, 1958).

Bowman (1932) found student preference for particular types of problems to be related to intelligence. Four hundred thirteen junior high school students were involved in the study. Students of high ability tended to have no distinct preference for the types of problems, and performed equally well on all of them. However, students of lower intelligence preferred problems involving computation only. They tended to dislike problems having complex situations or requiring descriptive analysis.

"It would appear that reading ability of students, reading level of materials, and vocabulary of both must be considered as being closely interrelated with learning to solve verbal problems" (PREP, 1972, p. 9).

Music

"From time to time such characteristics as a fondness for music and other arts . . . have been attributed to mathematicians" (Aiken, 1973, p. 24). In a study by Jenkins (1961), 256 high school students, selected by school counselors, participated in a study designed to examine the relationship between music aptitude, mental ability, math and science aptitudes. Half of the students had had no training in music, the other half not only has training but were currently in a school music organization. Mathematical aptitude was measured by the DAT Numerical test and musical aptitude was measured by the Drake Music Memory Test. Correlations between math and musical aptitudes ranged from 0.37 to 0.46. Crow and Crow (1965) found that gifted children have a high level of special aptitude in art, music, or science.

Socioeconomic status

The question of the influence of a person's background upon his achievement frequently occurs in studies on aptitude. "There is a large body of data showing a positive relation between occupational level and intelligence test performance" (Anastasi, 1966, p. 166). She has also concluded that "not only do urban groups excel rural groups on most intelligence tests, but these differences, too, are larger in verbal than in mechanical and spatial functions" (Anastasi, 1966, p. 166). In commenting about the instruments used to obtain these data, she states that tests may reflect the test constructor's social class, and that the samples used may have been biased toward urban subjects.

Sex

Most studies indicate the existence of sex differences in math achievement and attitudes toward math. In elementary school, differences in math achievement tend to favor the girls; the data are conflicting for junior and senior high school students (PREP, 1972).

In a study conducted by Burgert (1935), boys' math grades were higher than the girls' in ninth grade. Foran and O'Hara (1935) also found that boys scored higher than girls on a geometry test, independent of intelligence. In a study with 2450 elementary school students, Stroud and Lindquist (1942) reported girls to be superior to, but not significantly different from, boys in all school subjects except arithmetic. Sheehan (1968) also found girls to be superior to boys in ninth grade on a math problem solving test. When effects of previous achievement, aptitude, and IQ were removed, the boys were superior. Alexander

(1962) observed no significant differences between seventh grade boys and girls on a problem solving test, whether intelligence was controlled or not. "Taken collectively, the research results show that, on the average, girls tend to score higher than boys on tests of verbal ability, arithmetical fundamentals, and rote memory, whereas boys are superior in spatial ability, arithmetic reasoning, and problem solving" (Aiken, 1973, p. 17-18).

Differences in attitudes toward math may also be due to sex differences. Kauffman (1955) found boys' interests in math to increase with age. High school boys' attitudes were more positive toward math than the girls' (Farley, 1969).

Teacher ratings

It is generally believed that teachers should know their students well enough to be able to identify certain general learning characteristics such as math aptitude.

In Coy's study (1923), elementary school teachers were asked to select their five brightest students in their classroom. In comparing this selection to the selection via intelligence test scores, he found that the teacher rating did slightly increase the selection accuracy.

Terman (1925) also used teacher ratings of the youngest and the brightest children for his study involving 1528 students. This rating included an additional ten percent of the students who would have been excluded by the usual testing procedures. Thus, teacher ratings may be a helpful means of identifying specific characteristics of students.

Summary

In general, cognitive measures tend to account for close to fifty percent of the total variance of math achievement scores. Affective measures such as, personality and interest, do increase the predictability of the model but to a lesser extent. Math achievement has also been found to relate directly to other subject matter, e.g., science, reading, and music. Socioeconomic data indicate that disadvantaged students do not perform as well in mathematics as their peers. Boys have usually excelled in math and science, and girls have been found to perform better in more "verbal" courses of study.

Studies Related to Algebra and Geometry

Algebra and geometry are two different disciplines in mathematics. With modern math courses several attempts have been to integrate both approaches. Most of the existing studies on algebra and geometry concentrate on methods of teaching or course content; few studies have tried to identify those factors which indicate success within the course. Sill fewer investigate the differences between predictors of success in algebra and geometry.

Both algebra and geometry aptitude tests have shown some predictive validity. Hanna (1967) correlated final algebra grades with the Orleans-Hanna Prognosis Test and found the coefficient to be 0.70. Final grades in geometry were also correlated with the Orleans-Hanna Prognosis Test, yielding a coefficient of 0.60.

Students who had failed algebra and geometry were found to rank in the lowest half of their high school class; seventy percent of these were in the lowest quarter (Joseph, 1940).

Algebra

Dinkel (1959) attempted to answer the questions of who should take algebra and when it should be taken. Those variables which correlated highly with final algebra grades were arithmetic test scores and Orleans Algebra Prognosis Test scores, and past arithmetic grades. Variables which did not contribute to the prediction equation were: age, sex, parent's opinion about algebra, work habit grades, and reading grade placement.

Messler (1961) also found that age did not make a difference in success in algebra. His concern was the appropriateness of algebra for eighth grade students.

Intelligence test scores have also been found to be the best predictors for success in algebra (Baldauf, 1963; McCuen, 1930).

Barnes and Asher (1962) found that past achievement in math, as measured by eighth grade arithmetic grades, predicted success in algebra better than IQ, or scores from reading and algebra aptitude tests. Similar results were obtained by Mogull and Rosengarten (1971). The multiple correlation coefficient reached 0.50 with Math 8 grades but did not increase significantly with the additional variables (i.e., DAT Verbal, Numerical, and Abstract Reasoning tests, and Iowa Algebra Aptitude Test).

Eighth grade math grades were also found to be the best predictor of success in algebra in McQueen and Williams' study (1958). He also found that reading and IQ were not significant variables in the prediction of success in algebra.

To summarize, the best indicators of success in algebra have been: IQ, past achievement in math (usually Math 8 grades), and an algebra aptitude test; reading test score and age also correlate highly with algebra grades.

Geometry

Some variables which were found to be good predictors of success in algebra are also good predictors of success in geometry. Reading ability tends to be more highly correlated with geometry achievement than with algebra achievement (Bennett, Seashore & Wesman, 1952).

D'Augustine (1963) found reading and arithmetic achievement scores to be significant factors in achievement in topics of topology and geometry, but sex and grade level did not have a significant effect on the test scores.

Hanna and Lenke (1970) also investigated the relationship of final grades in geometry to age, sex, and the student's prediction of his grades. Grade prediction by those students who were realistic about their capabilities was the best single predictor of the final grades.

Three geometry prognosis tests (Iowa, Orleans, Lee) have been compared by Hanna (1967). Correlations ranged from 0.47 to 0.52 between the tests and final course grades, with the Orleans test being the best predictor. Teacher rankings of student's achievement also correlated highly with the final grades. Both algebra and arithmetic grades correlated in the low 0.20's with the final geometry grades. He also investigated the validity of interest inventory scales for the prediction of success in geometry (1966). Hanna believes that the value of these interest scales for prediction is modest.

Summary

What a student learns is dependent upon the unique patterns of aptitude possessed by the individual. To be able to predict success for each individual in specified courses, it is necessary to consider the interaction of a set of aptitudes with the treatment (courses). Research has indicated that success in mathematics is dependent upon the following set of variables: intelligence, personality, interest in mathematics, differential abilities, reading, music, socioeconomic status and sex. No one study has had a comprehensive investigation of all of these variables and their interactions with different mathematics courses. Although algebra and geometry are two separate disciplines (treatments) within mathematics, research has indicated differences in predictors of success in each. Intelligence, previous year's grades in math, and specific prognosis tests have tended to be strong predictors for both algebra and geometry. Reading ability may predict success in geometry better than for algebra.

Since the percent of total variance accounted for by predictors of algebra and geometry grades have not risen above fifty percent, it is possible that other aptitudes must play an important role in differential prediction. This study will investigate the interaction of those aptitudes which have been successful in predicting math achievement with two treatments, algebra and geometry.

METHODS

Subjects

Students selected for this study were juniors from the two public high schools in the sixth largest school district in Iowa. This system enrolled 15,105 students during the 1972-1973 school year (Williams & Warf, 1973). Two thousand eight hundred fifty-eight students attended high school, and 922 were juniors.

All juniors having taken both algebra and geometry were included in the target population (n = 371). Seventy-one students were eliminated from the study: 1) 28 students were not informed of the test; 2) 42 students' work-study schedules conflicted with the testing time; and 3) one student refused to identify himself on the test materials. Thus, 300 students were used in the analyses. Of these, 62% were from one high school and 38% from the other; 52% of the sample were males (see Table B.1).

The students came from families having an average education of 12½ years (see Table B.2). The parents' average occupational skill was that of a blue collar worker (see Table B.3).

The 71 students who were excluded from the sample were measured on socioeconomic status, parents' education level, math grades, standardized achievement tests, and IQ. There were no significant differences between this group and the experimental sample (see Table B.4).

The sample was divided twice to identify specified subgroups. The first division was for the purpose of looking at differences between successful and nonsuccessful math students. Eighty students who were rated by their former math teachers as having high mathematical aptitude (5.5 and above) and also having obtained a math grade point average between 3.00 and 4.00 comprised a math "stars" subgroup. Forty-six students met the criterion for math "nonstars" (math GPA less than 1.75 and teacher rating lower than 4.0) (see Table B.5).

The second division was made for the purpose of identifying aptitude-treatment interactions. Differences between the algebraic stars and geometric stars were examined. Algebraic stars were defined as students whose algebra GPA was greater than geometry GPA. There were 98 algebraic stars. One hundred twenty-four students were identified as geometric stars since their geometry GPA was greater than their algebra GPA (see Table B.6).

Instruments and Variables

Data were obtained from students' cumulative files, former mathematics teachers, and four instruments administered to the students. The variables studied, as well as the instruments employed to measure them, are described below.

1. Measures of success in mathematics, and algebra and geometry in particular, were obtained from the students' cumulative files. The Iowa Tests of Educational Development (ITED), which were administered to the students in ninth grade, were used as standardized measures of achievement. The ITED provided nine subscores: Basic Social Concepts, General Background in Natural

Science, Correct and Appropriate Expression, Quantitative Thinking, Interpretation of Social Studies Reading Material, Interpretation of Natural Science Reading Material, Interpretation of Literary Material, General Vocabulary, and Use of Sources of Information. Semester grades in algebra and geometry were also recorded for all students; tenth grade class rank was obtained for many of the students.

2. Aptitude measures included scores on the Multi-Aptitude Test (MAT, Cureton, Cureton et al., 1955) (see Appendix A.1); IQ as measured by the Otis-Lennon Mental Ability Test; and teacher ratings (TR) of the student's mathematical aptitude (see Appendix A.2). Six of the ten subtests of the MAT were administered; Vocabulary (verbal relations); General Information (verbal relations); Arithmetic (number factor); Number Series (general reasoning); Figure Classification (general intelligence); and Paper Form Board (spatial perception and visual orientation). The Otis-Lennon had been administered to the students in ninth grade.

Mathematics teachers were asked to rate the math aptitude of their former students in the sample on a 7-point scale. Twenty-six students were not rated; all other students were rated by as many as three teachers. When several teachers rated the same student, the median rating was used. Teachers were also asked to state whether the student's performance and aptitude matched or were incongruent. The distribution of the teacher ratings and math grade point averages is shown in Table B.5.

3. Demographic data consisted of the sex of the student and information about his socioeconomic status; these were secured from the student cumulative files. Race was deleted since there were only three non-Caucasian students in the sample. Both mother's and father's last school grade completed and their current occupations were used as indices of the student's socioeconomic status (SES). The occupational information was coded according to the U.S. Government Census Codes (1950).

4. Nine scales from the Edwards Personality Inventory (EPI), were used to assess personality characteristics potentially related to problem solving. The EPI was selected because "although the validity data do not appear in the manual, the EPI is an instrument worthy of serious consideration by those interested in the assessment of a broad range of personality characteristics in 'normal' adolescents and adults" (Norman, 1972, p. 154). The scales used were: Plans and Organizes Things, Intellectually Oriented, Persistent, Self-confident, Has Cultural Interests, Conforms, Seeks New Experiences, Likes to be Alone, and Is a Perfectionist. These were combined into an inventory of 190 true-false items (see Appendix A.3); and the student was directed to "predict how people who know you well would mark each statement if they were asked to describe you" (Goldberg, 1972, p. 151).

5. Information about student's musical experiences, both vocal and instrumental, was obtained from a questionnaire developed by the investigator (see Appendix A.4). Type of instrument(s) played, number of years, and type of instrumental or vocal group were used to identify the student's degree of musical experience. This information was included as it has been suggested (Crow & Crow, 1965) that special aptitudes in art, music, or science may be related to personality traits and a high level of general intelligence. Musical experiences were included in order to measure its potential relationship to mathematical aptitude.

6. Interest in mathematics, reading, algebra and geometry were measured by the Semantic Differential Interest Inventory (SDII; see Appendix A.5). This instrument is identical to one developed by McCallon and Brown (1971) for interest in mathematics. Their instrument correlated 0.90 with the Likert-scaled Math Attitude Scale (MAS) developed by Aiken (1972a). The semantic differential was chosen over the MAS due to its flexibility for expansion to include other concepts and because the "correct" answers would not be so obvious to the students.

   Reading, algebra and geometry were used as concepts and were rated on the same scales as for mathematics. Scales appeared in different positions from concept to concept in order to prevent response patterns.

7. As problem solving tests tend to be situation-specific, the

search for suitable, reliable problem solving tasks led back to specific test items. The five problems used in the final form of the Problem Solving Test (PST; see Appendix A.4) were gleaned from well over 200 tasks found in books (Gardner, 1961; Court, 1958) and studies (Garrett, 1970; Shaw, 1963). Shaw's set of group problem solving tasks was used as the nucleus for this study since his data contained an analysis of the 104 tasks on ten dimensions, e.g., item difficulty, clarity of directions, and type of task.

Validation of the Problem Solving Test

Seven tasks were originally selected for the trial form of the PST. Fifteen professors and graduate students from Iowa State University and two high school math teachers then reviewed the trial form, administrative directions, solutions and scoring for each problem. Raters represented the following departments: Mathematics, Psychology, Education, Statistics, Sociology, and Computer Science. Each rater evaluated each task on twelve concepts and on the difficulty of the item for high school juniors who have taken both algebra and geometry (see Appendix A.6). The raters viewed the problem solving test as measuring logical reasoning (items 2, 3, 4, 5), algebraic skills (items 1, 3), geometric skills (item 3), spatial perception (item 3), and accuracy (item 1). Only a moderate importance was placed on arithmetic skills, verbal and nonverbal ability, flexibility, persistence, application and trial and error.

Inter-rater agreement on the content of the Problem Solving Test, as measured by the average off-diagonal correlations, was quite strong, $\bar{r} = 0.42$ (see Tables B.8 and B.9). However, there was a great deal of disagreement between the judges when rating the difficulty of the items (see Tables B.10 and B.11) as shown by the average off-diagonal correlation being essentially zero ($\bar{r} = 0.08$).

The seven-item trial form of the PST was administered to 22 high school juniors who were currently enrolled in the second course in algebra. The 40 minute time period was a limiting factor on students' scores as few students reached the end of the test. The discrimination between students on problems 1 and 4 was poor since all of the students obtained full points for problem 1 and all but two students failed problem 4 (see Table B.11). After discussing the PST with these students and inspecting the item difficulty and reliability coefficients, problems 1 and 4 were deleted and the directions for problem 2 were clarified.

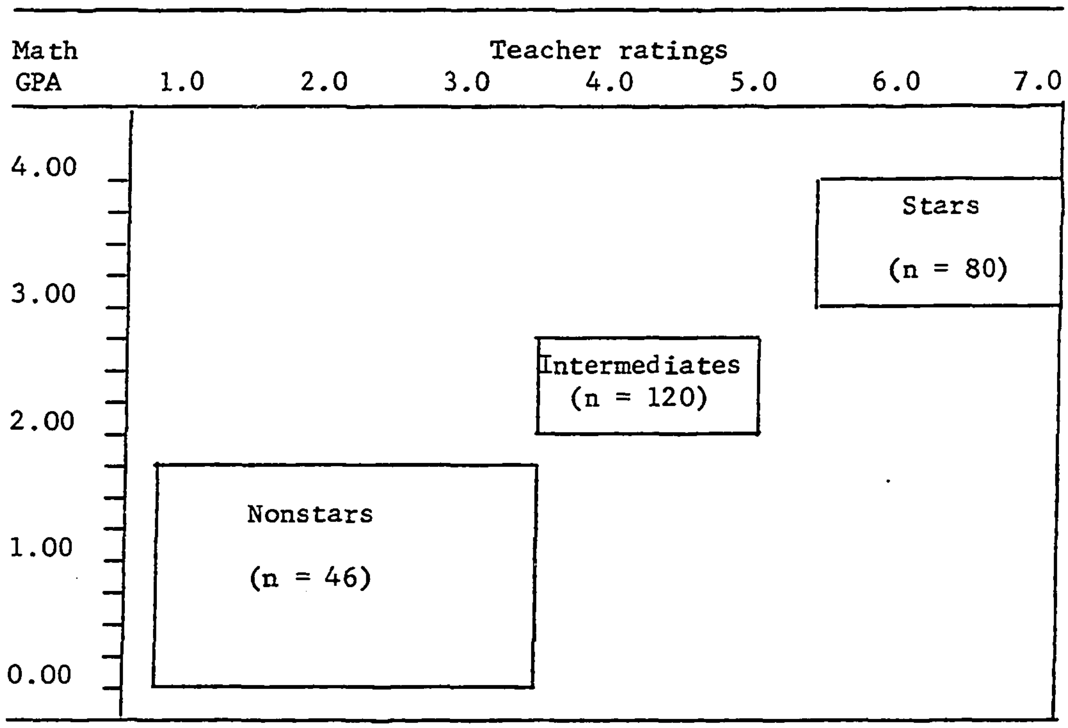
With the number of problems reduced from seven to five, and the time limit held at 40 minutes, a revised form of the PST was administered to another sample of 37 students. In order to determine the limits of the PST, a wider range of mathematical experience was sought; this sample was composed of twenty juniors currently enrolled in the second course in algebra, two seniors who had completed three semesters of mathematics beyond geometry, and 15 sophomores currently beginning their second semester in geometry. The correlation matrix and reliability coefficient are given in Table B.12. With this sample, forty minutes was ample time for students to complete the test. The reliability coefficient increased from 0.53 to 0.81 and the average item difficulty changed from $\bar{p} = 0.36$ to $\bar{p} = 0.43$. The decision was then made to retain the revised form of the Problem Solving Test as the final form of the instrument.

Design and Analysis

Since there were two main purposes of this study, the design was structured differently for each part. The first purpose was to determine which factors constitute mathematical aptitude, and what patterns of variables differentiated between high and low ability math students. In Design I (see Table 2) the sample was divided into three subgroups (stars, intermediates, and nonstars) based on performance in math courses and teacher ratings on the student's math aptitude. Inter-group differences on differential abilities, problem solving ability, subject matter interest factors, musical experiences, achievement scores, IQ, socioeconomic status and sex were then tested.

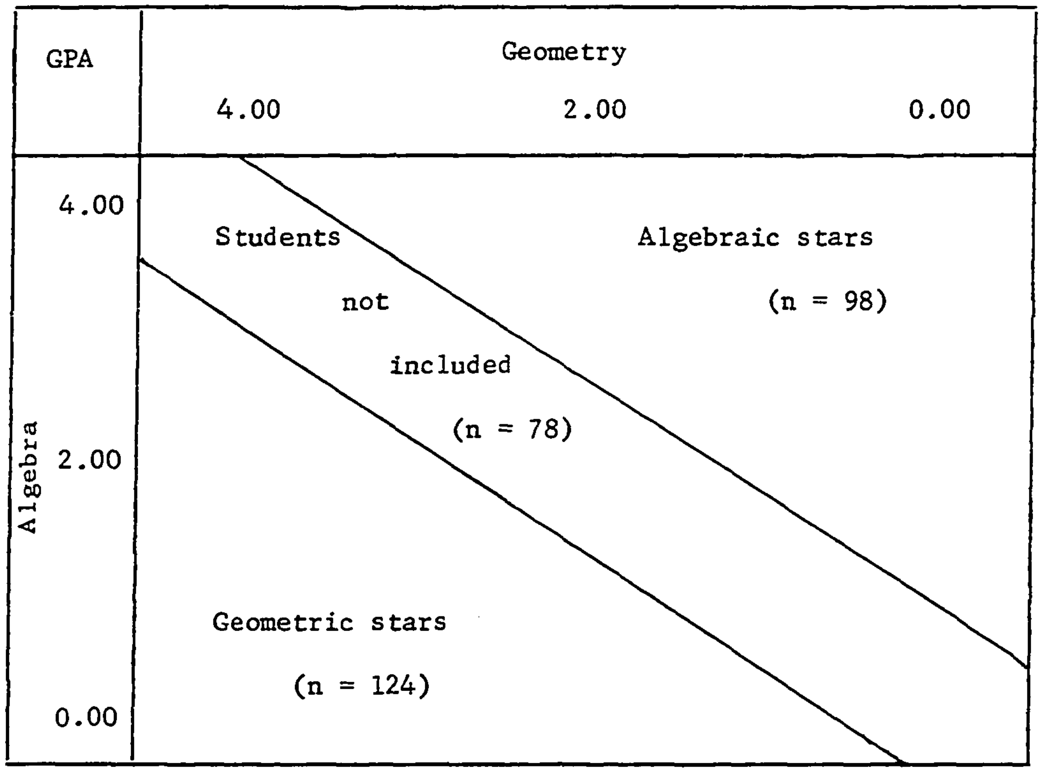
To assure that the proportion of number of variables to sample size did not exceed the limits of the statistical methods used (Nunnally, 1967), factor analyses were used to group and reduce the number of variables. Factor scores of the personality inventory and of the interest inventory were placed in a stepwise regression along with the remaining set of variables. Star, intermediate, and nonstar means were compared by multivariate analysis of variance. To determine where the significant differences were, separate analyses of variance were computed, and the means of those variables producing significant differences were plotted.

Table 2. Design I: Stars and nonstars



Design II (see Table 3) examined the differences between algebraic stars and geometric stars, the purpose being to determine whether any interactions existed between the variables used in the stepwise regression model from Design I and the treatments, algebra and geometry. A multivariate analysis of variance indicated significant differences between treatments. After testing for equal variances separate t-tests were computed. Means of variables producing significant differences were also graphed.

Table 3. Design II: Algebraic stars and geometric stars



Data Collection

Data were collected from the students' cumulative files, from math teachers and from four tests which were administered to the students by the investigator. The tests were administered at the two schools on the same day, during a two hour test period. Tests which were partially completed were individually readministered during the following week by a school counselor. Math teachers at both schools individually rated their former students on their mathematical aptitude, using a rating form and listing provided by the investigator (see Appendix A.2).

Assumptions and Limitations

Since research on mathematical abilities is so recent and incomplete (Romberg, 1969; Cronbach, 1967), it is quite possible that not all variables associated with the learning of mathematics have been identified. In this study it is assumed that the principal factors involved in mathematical aptitude have been taken into account. The instruments used were assumed to be valid measures of these variables and that the responses on the personality (EPI) and interest inventories (SDII) were assumed to be true reports of a student's behavior and feelings. It was also assumed that a student's personality remained relatively constant during the two years of participation in the algebra and geometry courses. Other assumptions included: testing conditions were such that systematic errors were kept to a minimum; interest in mathematics and reading at the time of testing were similar to the interests held by the student prior to and during his participation in the algebra and geometry courses; and the sample was representative of the population of students from similar schools who have taken both algebra and geometry.

The accuracy of the results and the implications from the data are restricted by the reliability and validity of the instruments employed, the representativeness of the sample, and the limitations of the statistical procedures. The major methodological limitations include the following: correlation does not prove causation; different methods of factor analysis may result in different relationships and thereby lead to different conclusions; and the combination of items within a factor computed from factor analysis methods may not be readily interpretable. In addition, the design does not allow separation of teacher effects from the overall treatment effects.

RESULTS

Most of the data collected from the student cumulative files were complete. For some individuals a few scores from the ITED subtests, teacher ratings, or parents' education levels were missing. In each case, sample means were computed and inserted for these missing values. Mean responses were also supplied for those unmarked items on the EPI and SDII inventories. Sex was coded: 1 = female, 0 = male. The variables VOCAL and INSTR (instrumental), represent the number of years of experience the student had had.

Data Reduction

EPI

In order to reduce the ratio of variables to the number of subjects, the EPI scales were factor analyzed. An unrestricted maxium likelihood factor analysis solution with varimax rotation was used. Three factors were found (see Tables 4a and 4b). Tucker's reliability coefficient of 0.86 was obtained for these factors where $\chi^2 = 42.75$ (df = 12, $p < .005$).

Inspection of the three factors indicated a need for further rotation of factors 1 and 3. The final set of factor loadings for the EPI were obtained through a rotation of cos 45^{o} (see Table 5). Scales loading on factor 1 were: does not seek new experiences, is intellectually oriented, and likes to be alone. A student scoring high on this factor could be described as independent, intellectually oriented person who

Table 4a. EPI scale intercorrelations[a]

Scales (n = 300)										$\bar{X}$	S
Plans and organ.	1	-								9.69	4.39
Intell. oriented	2	24	-							16.16	5.58
Persistent	3	44	50	-						10.51	3.77
Self-confident	4	20	43	40	-					9.88	3.34
Cultural inter.	5	16	50	26	18	-				7.60	3.65
Conforms	6	27	-15	-01	-20	-19	-			9.18	3.51
Seeks new exp.	7	-01	08	08	09	26	-33	-		12.02	3.25
Alone	8	04	35	28	25	27	-15	-12	-	7.32	3.02
Perfectionist	9	57	43	54	23	20	16	-06	08	13.85	5.24
Unique variances		.43	.47	.72	.65	.67	.01	.72	.38		

[a]Decimals omitted.

Table 4b. EPI factor loadings

		Varimax-rotated factor loadings[a] 1	2	3	h^2
Plans and organizes	1	-.00	.75	.06	.566
Intellectually oriented	2	.07	.29	.77	.682
Persistent	3	.08	.55	.47	.753
Self-confident	4	.08	.18	.49	.279
Cultural interests	5	.26	.12	.52	.352
Conforms	6	-.32	.35	-.33	.324
Seeks new experiences	7	.997	-.01	.01	.980
Likes to be alone	8	-.13	-.02	.52	.288
Perfectionist	9	-.05	.74	.27	.623

[a]χ^2 = 42.751, df(12); Tucker's Reliability = 0.860.

Table 5. Rotated EPI factor loadings

	IW	OP	INC	h^2
1	.04	.75	.04	.566
2	.49	.29	.59	.672
3	.28	.55	.38	.525
4	.29	.18	.40	.277
5	.18	.12	.55	.572
6	-.01	.35	-.46	.324
7	-.70	-.01	.71	.994
8	.45	-.02	.28	.281
9	.23	.74	.15	.623

usually withdraws from social activities. This factor will be referred to as EPI-IW where IW represents intellectual-withdrawn. Scales loading on the second factor included: plans and organizes things, is a perfectionist, and is persistent. A high score on this scale indicates personal drive, emphasis on organization and attention to details. This scale will be referred to as EPI-OP where OP represents organizer-perfectionist. Five scales loaded on the third EPI factor: seeks new experiences, is intellectually oriented, has cultural interests, is a nonconformist and has self-confidence. Students with high scores on this factor tend to be nonconforming intellectuals who have broad interests. This factor will be referred to as EPI-INC where INC represents intellectual-nonconformist.

EPI factor scores were obtained from the product of $XR^{-1}F$ where X was the raw score matrix, R^{-1} was the inverse correlation matrix, and F was the factor loadings matrix. Since many scores had negative values, a linear transformation was of the form: $X = 10(x+5)$; and for the factors, EPI-OP and EPI-INC the form was $X = 10(x+10)$. These values made interpretation easier and provided greater mathematical accuracy. These three factor scores for each individual were used in further analyses.

SDII

Each of the four concepts-mathematics, reading, algebra, geometry-contained the same fifteen scales, thus yielding a total of 60 items to be factor analyzed (see Table B.17). The maximum allowable factors, 19 (4 concepts and 15 scales), were extracted by an unrestricted maximum likelihood factor analysis solution with varimax rotation. From inspection of the difference in the latent roots a practical solution of eight factors was indicated. The response scores were again factor analyzed and rotated. Seven of these factors had loadings high enough to be interpretable (see Table 6). Factor scores were computed from the loadings on these seven factors and were used in subsequent analyses.

The first four factors were subject-matter specific, that is, high scores on SDII-A indicated high interest in algebra, SDII-G interest in geometry, SDII-M interest in mathematics, and SDII-R interest in reading. The last three scales reflected students' feelings toward math, algebra, and geometry. A high positive score on SDII-EZ indicated that the student felt math, algebra and geometry were "soft-light" subjects. SDII-MOT factor represents the intrinsic motivation of the subject. A high score indicated the student's feeling that math is "active-varied"

Table 6. Factor loadings from the SDII

	A	G	R	M	EZ	MOT	VAL
MATHEMATICS							
pleasant	.06	.01	-.07	.41	.06	.09	-.05
good	-.17	-.04	.01	.32	.02	.05	-.19
active	.08	-.00	-.01	.24	-.09	.19	-.10
valuable	-.10	-.01	.11	.38	-.10	.09	-.26
strong	-.08	-.13	.08	.25	-.04	-.01	-.14
love	.14	.05	.04	.36	-.11	.07	-.02
fast	-.01	-.11	.03	.13	-.01	.00	.06
comfortable	-.02	-.03	.03	.37	.04	.03	.06
nice	.04	.05	.03	.44	-.09	.12	.07
enjoy	.09	.00	.00	.44	.00	.05	.00
varied	-.04	-.02	.02	.06	-.03	.67	.03
soft	-.09	-.07	-.06	.19	.47	-.07	.08
unafraid	-.05	-.06	.01	.05	.01	-.02	.40
light	-.06	-.05	.08	.04	.56	.09	.11
secure	-.01	-.15	.04	.15	-.01	-.02	.18
READING							
pleasant	.02	.05	.86	.07	-.01	.15	.11
good	-.02	-.02	.81	.10	-.09	.10	-.11
active	.01	.02	.49	.05	-.10	.20	-.00
valuable	-.04	.09	.62	.12	-.24	.15	-.10
strong	-.00	-.03	.80	.04	-.03	-.04	.02
love	.05	.01	.80	.09	-.01	.15	-.05
fast	.08	-.10	.72	-.04	-.10	.00	.08
comfortable	-.04	.04	.85	.09	-.06	.05	-.02
nice	-.04	.02	.85	.04	.01	.15	-.10
enjoy	-.04	.00	.77	.17	-.06	.09	-.13
varied	.13	.03	.49	.05	-.12	.19	-.01
soft	.06	-.15	.69	.03	.06	-.01	.07
unafraid	-.00	-.05	.52	.04	-.15	.02	.15
light	-.14	-.04	.40	-.00	.21	.03	.06
secure	.06	-.12	.79	.08	-.08	.12	-.01

Table 6 (Continued)

	A	G	R	M	EZ	MOT	VAL
ALGEBRA							
pleasant	.46	-.10	-.07	.08	.01	.17	-.11
good	.34	-.18	-.13	.10	-.04	.18	-.14
active	.26	-.06	-.09	.04	-.10	.20	-.18
valuable	.26	-.09	-.07	.15	-.04	.22	-.38
strong	.19	-.20	-.15	-.03	-.11	.06	-.07
love	.53	-.10	-.10	.13	-.09	.20	-.11
fast	.16	-.17	-.13	-.08	.11	.05	-.08
comfortable	.34	-.13	-.08	.07	.04	.12	-.01
nice	.38	-.05	.12	.11	-.01	.24	-.10
enjoy	.41	-.14	-.12	.14	.03	.18	-.15
varied	.06	-.09	-.02	.01	.03	.63	-.03
soft	.22	-.21	-.09	-.03	.63	.02	.01
unafraid	.21	-.20	-.06	.06	-.02	-.05	.20
light	.13	-.21	-.10	-.04	.72	.05	.04
secure	.41	-.24	-.06	-.00	-.06	.07	.05
GEOMETRY							
pleasant	.00	.52	-.01	.04	-.03	.12	.12
good	-.02	.43	-.00	-.01	.19	.11	-.03
active	.02	.34	.03	-.04	-.11	.13	-.08
valuable	.03	.30	.18	-.04	-.06	.20	-.25
strong	-.19	.32	.03	-.10	.01	-.10	.15
love	.05	.50	.04	.02	.00	.12	.06
fast	-.06	.29	.04	-.17	.05	-.02	.08
comfortable	-.06	.42	.05	-.02	.12	.01	.17
nice	.05	.52	.01	.02	.09	.17	.05
enjoy	.07	.46	.00	-.00	.04	.14	.08
varied	-.02	.10	.00	-.07	.01	.41	-.03
soft	-.13	.33	.06	-.07	.73	-.06	.13
unafraid	-.16	.26	.05	-.11	.02	.07	.34
light	-.07	.21	.11	-.09	.77	.01	.00
secure	-.09	.35	.08	-.08	-.00	.00	.24

but a low score meant that he felt math was boring, i.e., "passive-repetitive". The seventh factor, SDII-VAL, was a type of evaluative-anxiety factor. Those students with high scores felt math was valuable but had some feelings of insecurity about the courses. A low score on the other hand, indicated that the student found math to be worthless and was therefore secure in it.

Interrelationships of Variables

Correlation coefficients showing the interrelationships of all personality, standardized math and verbal test scores, interests, musical experience, sex, socioeconomic status, teaching ratings and grades, and problem solving measures are shown in Table B.18. ITED subtests, math grades, teacher ratings, and IQ tend to have high positive correlations with other variables and music, sex, and socioeconomic data show near zero correlations. Personality, problem solving and interest correlation coefficients had a tendency to fluctuate between ± 0.35.

Design I. Factors of Mathematical Aptitude

Three stepwise regressions were computed in order to determine which variables contributed significantly to the prediction of success (GPAs) in algebra, geometry, and mathematics. A fourth stepwise regression was run to find out which variables predicted teacher ratings of students' mathematical ability. Table 7 displays those variables which contributed .01 or more to R^2. The variables are shown in the order of entrance into the regression equation. The complete set of significant

Table 7. Order in which significant predictors entered the regression

Dependent variable							
Algebra		Geometry		Mathematics		Teacher rating	
Variable	R^2	Variable	R^2	Variable	R^2	Variable	R^2
QUANT	.16	IQ	.33	QUANT	.29	QUANT	.25
MARITH	.19	EXPR	.38	MARITH	.34	MARITH	.34
SEX	.21	MARITH	.43	USES	.38	LIT	.38
EPI-OP	.23	NSREAD	.45	EPI-OP	.40	BUS	.40
NSREAD	.25	EPI-OP	.46	NSREAD	.42	EPI-OP	.42
VENN	.26	BUS	.48	SEX	.44	EQUA	.43
MFIG	.27	--	--	MFIG	.45	--	--
SDII-M	.29	--	--	SDII-M	.46	--	--
SDII-G	.30	--	--	SDII-EZ	.47	--	--
EPI-IW	.31	--	--	--	--	--	--

predictors of each regression can be found in Tables B.13-16.

Y = Algebra grades

Interpretation of the regression analysis alone shows that the best predictor of grades are computational skill and quantitative thinking ability. The successful algebra student also needs to be persistent, organized and a perfectionist; and he must be able to manipulate numbers and figures, as in a jig-saw puzzle, and be able to interpret natural science readings (see Table B.13). High interest in mathematics and in geometry are also characteristic of successful algebra students. These students may also tend to be intellectuals who withdraw from new and social activities. This regression accounted for only 36% of the variance in algebra grades which indicates other potent variables may have been excluded or that the measurement error was quite large.

Y = Geometry grades

Success in geometry depends primarily upon the student's general intelligence, verbal and computational abilities. Ability to comprehend natural science reading material and to solve simple topological problems seems to facilitate learning of geometry (see Table B.14). A successful student of geometry also needs to be persistent, organized and a perfectionist. Grades in geometry were predicted more accurately than algebra grades--58% of the variance in geometry grades was accounted for in the regression.

Y = Math grades

Overall success in mathematics, as determined by the average of semester grades in algebra and geometry, had several similar and a few different predictors of success than in algebra or geometry alone (see Table B.15). Mathematical success depends upon computational ability and quantitative thinking; this result is similar to the algebra predictors. A student must have an ability to apply information and have a feeling that math is a "soft-light" subject. For success in math a student must be organized and persistent as well as a perfectionist. He must also have an ability to interpret natural science readings and have a high level of spatial perception. High interest in math is also indicative of success. This regression accounted for 54% of the variance in math grades.

Inspection of the zero-order correlations between math grades and interest showed no relationship existed; however, interest in math had a relatively important position in relation to other variables in the stepwise regression predicting success in math. In contrast, IQ had a high zero-order correlation with grades but did not contribute much to the prediction of success in math or algebra after quantitative, personality and interest measures were added.

Y = Teacher rating

Math teachers rated their students on the amount of math aptitude they thought the students possessed. This rating can be best predicted by: student's computational skills and quantitative thinking; ability to interpret literature; being organized, persistent and a perfectionist; and ability to solve algebraic equations and topological problems (see Table B.16).

Stars vs. nonstars

The question arose as to whether the statistical significance of predictor variables was due to the characteristics of the total sample or whether the variables discriminated among groups of students with varying degrees of success in mathematics. Thus, further analyses were run to determine which variables aid in the prediction of success in mathematics for the entire model and also remain significant predictors between star and nonstar students.

Students were divided into three groups according to their math grades and teacher ratings (see Table 2). There were 80 stars, 46 nonstars and 120 intermediates. This grouping did not include the entire sample since 54 students obtained a high math GPA and low teacher rating, or a low math GPA and high teacher ratings.

A multivariate analysis of variance was computed in order to determine whether or not a significant difference existed among the three groups on all of the thirty-eight variables considered jointly. Pillai's trace equaled 0.74 which yielded an F value of 3.20 with 76 and 412 degrees of freedom. This statistic shows the existence of significant differences amount the groups at the $\alpha = 0.0001$ level.

Since this technique only determines the overall significance of the groups' contribution to the regression mean square, further inspection of the variables was made. One-way analysis of variance was computed separately for each of the variables. The F test for each ANOVA is shown in Table 8b. Those variables which had a significant difference among groups favored the star group in every case. These interactions are depicted in Figure 1. As can be seen from the figure, differences were between all three groups rather than between star or nonstars and the intermediate group.

The ITEDs, MAT, PST, and IQ all produced significant differences among groups, most of which were significant beyond the 0.001 level. Those variables which were found to contribute significantly to the overall prediction of success in math, algebra and geometry and also differed significantly among groups in favor of the star group are said to be factors of mathematical aptitude. These factors and the instrument used to measure them are: quantitative thinking (ITED), arithmetic skills (MAT), natural science reading (ITED), vocabulary (ITED, MAT),

Table 8a. Legend for variables in study

Variable	Variable name	Variable abbreviation
MAT	Vocabulary	MVOC
	General Information	MGENFO
	Arithmetic	MARITH
	Number Series	MNOSR
	Figure Classification	MFIG
	Paper Form Board	MPFB
ITED	Basic Social Concepts	SCON
	General Background in Natural Science	NSBK
	Correct and Appropriate Expression	EXPR
	Quantitative Thinking	QUANT
	Interpretation of Social Studies Reading material	SSREAD
	Interpretation of Natural Science Reading material	NSREAD
	Interpretation of Literary material	LIT
	General Vocabulary	VOCAB
	Use of Sources of Information	USES
OTIS-LENNON	--	IQ
PST	Missing Values	EQUA
	School Location and Bus Routes	BUS
	Ant Wants the Food	ANT
	Class Count	VENN
	The Fox, the Goose, and the Bag of Corn	FOX
SES	Mother's education	MED
	Father's education	FED
	Mother's occupation	MOCC
	Father's occupation	FOCC
EPI	Intellectual-withdrawn	EPI-IW
	Organizer-perfectionist	EPI-OP
	Intellectual-nonconformist	EPI-INC
Music	Instrumental	INSTR
	Vocal	VOCAL
SDII	Interest in algebra	SDII-A
	Interest in geometry	SDII-G
	Interest in mathematics	SDII-M
	Interest in reading	SDII-R
	Math is "soft-light"	SDII-EZ
	Math is "active-varied"	SDII-MOT
	Math is "valuable-insecure"	SDII-VAL
GRADES	Algebra-Semester 1	AL1
	Algebra-Semester 2	AL2
	Geometry-Semester 1	GE1
	Geometry-Semester 2	GE2
Rating	Teacher rating	TR

Table 8b. Mean differences among groups on predictor variables

Variable	Stars n = 80 X̄	S	Intermediates n = 120 X̄	S	Nonstars n = 46 X̄	S	F(2,220)
MVOC	7.15	2.74	5.59	2.46	4.78	2.21	15.32***
MGENFO	7.31	3.09	5.74	2.84	4.83	2.72	12.40***
MARITH	7.11	1.84	6.03	1.74	4.89	1.65	24.13***
MNOSR	7.21	1.96	6.49	1.73	5.09	2.05	18.91***
MFIG	10.68	2.12	10.34	2.16	9.24	2.71	6.13**
MPFB	4.56	1.99	3.64	2.26	3.57	2.08	5.23**
SEX	0.51	0.50	0.51	0.50	0.41	0.50	0.70
IQ	121.56	10.24	111.29	8.87	105.44	7.77	52.38***
SCON	19.63	4.30	16.42	4.11	13.74	4.42	30.19***
NSBK	20.70	4.34	18.45	4.16	15.07	5.78	22.37***
EXPR	19.98	3.25	16.67	2.66	14.85	3.71	47.42***
QUANT	21.94	5.19	17.26	4.45	12.61	4.81	58.11***
SSREAD	21.73	4.91	17.33	5.32	13.50	4.89	40.12***
NSREAD	22.64	5.14	18.37	5.26	13.72	5.46	43.10***
LIT	20.71	4.65	16.68	4.69	12.22	5.52	46.11***
VOCAB	20.45	4.19	17.51	3.44	15.78	3.76	25.85*
USES	21.84	4.01	18.39	4.05	14.54	5.39	42.68*
MED	12.48	1.53	12.19	1.32	12.26	1.47	0.97
FED	13.29	2.02	12.88	2.01	12.44	1.54	2.92

*α = .05 F = 3.07 df(2,120).

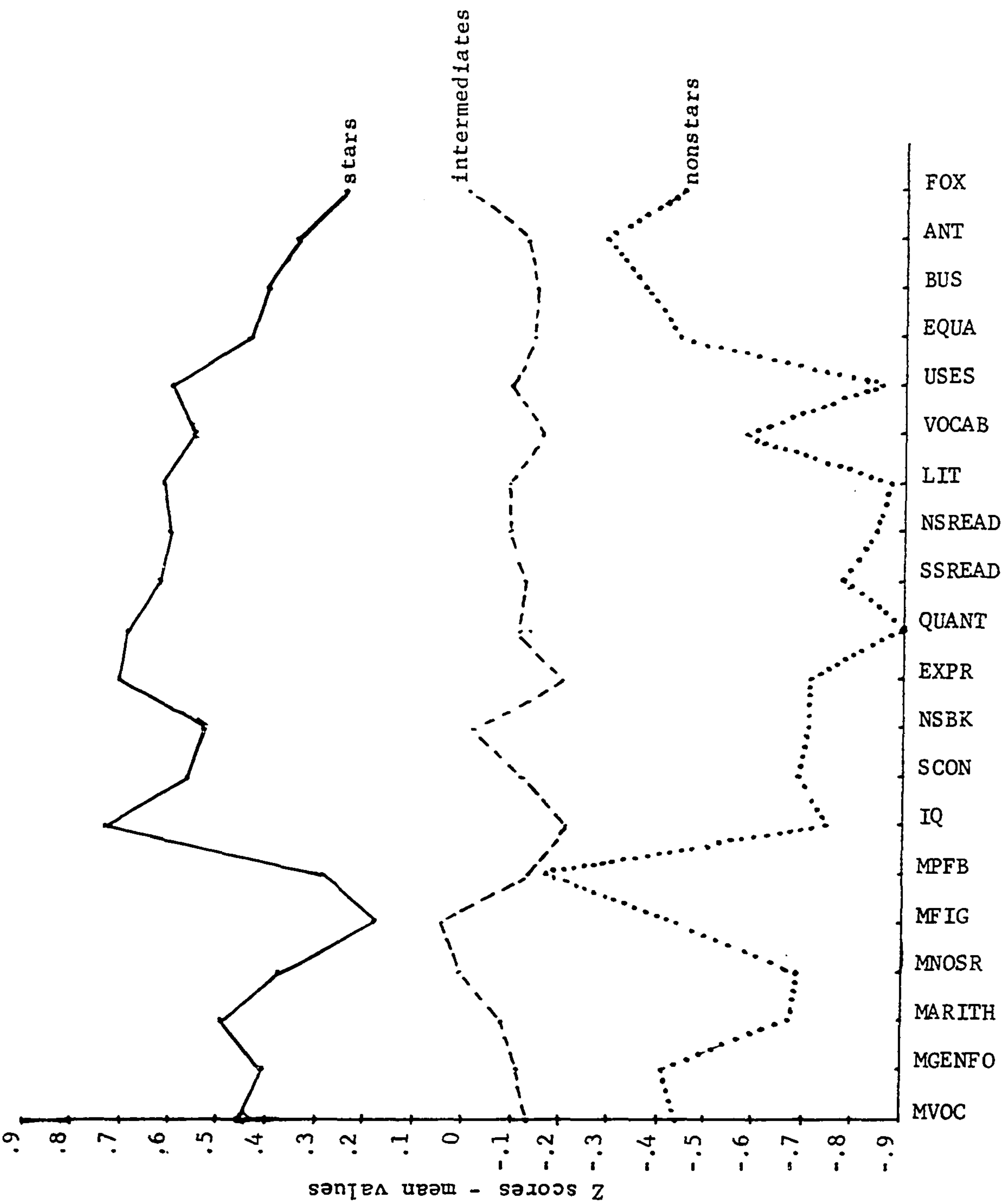
**α = .01 F = 4.79 df(2,120).

***α = .001 F = 7.32 df(2,120).

Table 8b (Continued)

	Stars n = 80		Intermediates n = 120		Nonstars n = 46		
	$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S	F(2,220)
MOCC	21.70	35.55	16.11	30.65	25.22	35.78	1.47
FOCC	59.23	26.05	65.23	23.90	59.67	25.90	1.68
INSTR	0.99	1.03	0.93	1.08	0.76	1.06	0.68
VOCAL	0.90	0.89	0.78	0.98	0.50	0.86	2.72
EQUA	0.98	1.10	0.43	0.88	0.13	0.54	14.80***
BUS	1.90	1.01	1.25	1.23	0.98	1.15	11.76***
ANT	1.83	0.79	1.41	0.86	1.26	1.02	7.97***
VENN	0.26	0.85	0.11	0.55	0.00	0.00	2.90
FOX	2.06	1.34	1.71	1.44	1.07	1.44	7.32**
EPI-IW	51.36	9.99	49.27	8.31	48.47	9.65	1.86
EPI-OP	100.84	8.22	99.85	7.52	98.54	6.31	1.37
EPI-INC	100.30	10.55	100.35	9.03	98.59	8.14	0.65
SDII-A	4.99	3.74	4.61	5.16	5.88	4.99	1.21
SDII-G	85.71	72.51	82.89	69.38	60.84	108.38	1.68
SDII-M	29.23	73.04	12.29	77.38	-5.17	88.72	2.92
SDII-R	158.78	73.30	165.84	103.41	128.12	93.35	2.79
SDII-EZ	21.86	27.01	16.46	27.00	15.56	26.60	1.21
SDII-MOT	0.62	3.84	-0.06	4.97	0.86	5.14	0.85
SDII-VAL	145.34	76.03	151.87	94.03	137.03	101.29	0.47

Figure 1. Variables which discriminate between stars and nonstars


stars
intermediates
nonstars
FOX
ANT
BUS
EQUA
USES
VOCAB
LIT
NSREAD
SSREAD
QUANT
EXPR
NSBK
SCON
IQ
MPFB
MFIG
MNOSR
MARITH
MGENFO
MVOC
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
Z scores - mean values

general information (ITED, MAT), general intelligence (IQ-Otis-Lennin, Figure Classification-MAT), verbal ability (Social Science Reading-ITED, Literature-ITED), spatial perception (Ant-PST, Paper Form Board-MAT), general reasoning and logical thinking (Bus-PST, Fox-PST, Number Series-MAT), knowledge of science (Natural Science Background-ITED), social concepts (ITED), and algebraic skills (Equation-PST).

All of the above variables can be considered factors contributing to success in algebra and geometry, but the order of importance among the variables changes with different criteria. There are three exceptions to the above statement. Although IQ, Literature subtest, and Fox item from the PST produced significant differences between groups, Fox was not a good predictor for geometry grades, and Literature and IQ were not significant predictors of algebra grades (see Table 9). These results are due in part to the partialling effects from the stepwise regression procedure. That is, variables are selected individually to enter the regression dependent upon all of the variables entered previously. The ITED subtest, Correctness of Expression, was a significant predictor of success in algebra and in geometry but not for the overall math average. The Vocabulary subtest of the MAT was a significant predictor of success in geometry only.

Design II. Aptitude-Treatment Interaction

Success in mathematics, as determined by algebra and geometry grades, is dependent upon the mathematical aptitude of the student. The factors of math aptitude are a composite of many personological variables

Table 9. Relative contribution to regression and order of predictors

Variable	Mathematics R^2 change	Mathematics Rank order	Algebra R^2 change	Algebra Rank order	Geometry R^2 change	Geometry Rank order
MVOC	--	--	--	--	.001	37
MGENFO	.001	35	.001	34	.001	30
MARITH	.048	2	.030	2	.042	3
MNOSR	.007	11	.004	15	.003	19
MFIG	.015	7	.013	7	.007	14
MPFB	.001	26	.002	22	.001	34
SEX	.015	6	.027	3	.004	17
IQ	.002	24	--	--	.328	1
SCON	.002	23	.001	29	.007	9
NSBK	.002	19	.002	20	.001	31
EXPR	--	--	.001	27	.057	2
QUANT	.288	1	.157	1	.006	10
SSREAD	.002	21	.001	32	.006	15
NSREAD	.018	5	.018	5	.019	4
LIT	.001	29	--	--	.003	22
VOCAB	.001	25	.002	21	.002	23
USES	.040	3	.001	24	.008	7
MED	--	--	.001	23	.001	28
FED	.003	15	.001	33	.004	18
MOCC	.002	22	.001	28	.002	24
FOCC	.001	34	.001	26	.001	36
INSTR	.001	30	.001	31	.001	29
VOCAL	.001	28	.008	12	.001	32
EQUA	.003	14	.003	17	.002	26
BUS	.001	27	.001	25	.011	6
ANT	.001	33	.002	18	.001	38
VENN	.009	10	.012	6	.001	27
FOX	.002	17	.002	19	.001	33
EPI-IW	.007	12	.010	10	.003	20
EPI-OP	.026	4	.016	4	.021	5
EPI-INC	.002	18	.001	35	.005	16
SDII-A	.008	13	.003	16	.002	25
SDII-G	.001	32	.011	9	.008	13
SDII-M	.011	8	.014	8	.008	12
SDII-R	.001	31	.005	13	.001	35
SDII-EZ	.012	9	.004	14	.006	11
SDII-MOT	.002	16	.001	30	.007	8
SDII-VAL	.002	20	.007	11	.003	21

and their interactions. Most of these variables can be found through standardized achievement tests involving verbal, mathematical, and problem solving skills as well as measures of spatial perception and general reasoning ability. The intereaction of these factors of math aptitude with the treatments, algebra and geometry, is important to educators in order to better individualize instruction and to produce better means of obtaining success. Design II was structured so as to enable an inspection of the interaction between mathematical aptitude factors and the treatments, algebra and geometry.

Students were divided into three groups according to their relative performance in algebra and geometry (see Table 3). There were 98 algebraic stars, 124 geometric stars, and 78 students with equivalent grades. A multivariate analysis of variance was computed in order to determine whether or not a significant interaction existed among treatments (groups of students). Pillai's trace equaled 0.39 which yielded an F value of 1.64 with 76 and 520 degrees of freedom. This statistic showed the existence of significant differences among the groups at the $\alpha = 0.0013$ level.

Since the purpose of this design was to determine interactions of aptitudes with the treatments, algebra and geometry, further analyses of these two groups was warranted. An investigation as to where the differences between treatments were occurring involved separate F and t-tests for each variable. Table 10 shows the means and standard deviations of the variables for treatments, as well as the t-test values. Most of the same items and subtests which were significant

Table 10. Mean differences between treatments on aptitude measures

Variable	Algebraic stars n = 98 $\bar{X}$	S	Geometric stars n = 124 $\bar{X}$	S	t df = 220
MVOC	5.37	2.46	6.50	2.68	-3.24***
MGENFO	5.20	2.96	6.51	2.99	-3.24***
MARITH	5.68	1.97	6.10	1.75	-1.65
MNOSR	5.91	1.95	6.41	1.98	-1.90
MFIG	9.68	2.58	10.17	2.29	-1.48
MPFB	3.57	2.26	3.88	2.25	-1.01
SEX	0.51	0.50	0.48	0.50	0.39
IQ	108.95	8.37	114.83	10.19[a]	-4.72***
SCON	15.41	3.66	17.60	4.94[a]	-3.79***
NSBK	17.33	4.12	19.06	5.23[a]	-2.76**
EXPR	15.76	3.48	17.68	3.54	-4.05***
QUANT	15.68	4.64	18.28	5.37	-3.80***
SSREAD	15.56	5.37	18.50	6.06	-3.77***
NSREAD	16.49	5.41	19.14	6.54	-3.23***
LIT	15.19	5.10	18.04	5.68	-3.88***
VOCAB	16.97	3.81	19.15	4.30	-3.95***
USES	16.79	4.72	19.23	5.31	-3.57***
MED	12.08	1.51	12.41	1.40	1.16
FED	12.47	1.99	13.10	1.94	-2.37*
VOCAL	0.56	0.84	0.84	0.97	-2.25*
EQUA	0.33	0.72	0.57	0.93[a]	-2.23*
EPI-IW	47.16	7.56	50.59	10.13[a]	-2.89**
MOCC	19.80	32.50	17.70	32.42	0.48
FOCC	59.88	26.23	64.02	23.35	-1.24
INSTR	0.84	1.01	0.93	1.03	-0.66
BUS	1.31	1.17	1.53	1.25	-1.38
ANT	1.47	0.91	1.68	0.87	-1.73
VENN	0.19	0.73	0.12	0.59	0.82
FOX	1.55	1.48	1.74	1.40	0.98
EPI-OP	98.85	6.98	99.55	7.87	-0.69
EPI-INC	100.89	8.24	99.69	9.83	0.97
SDII-A	4.85	5.52	4.73	3.77	0.20
SDII-G	66.49	83.68	96.79	77.49	-2.79***
SDII-M	13.03	86.94	6.41	71.37	0.62
SDII-R	165.58	88.29	145.87	92.19	1.61
SDII-EZ	17.82	27.48	16.71	26.52	0.31
SDII-MOT	-0.52	4.87	0.60	3.99	-1.89
SDII-VAL	146.57	84.24	141.27	86.49	0.46

[a]Unequal variance, separate t-test used.

*$t_{120} = 1.980$, $(p < .05)$; $t_{\infty} = 1.960$, $(p < .05)$.

**$t_{120} = 2.617$, $(p < .01)$; $t_{\infty} = 2.576$, $(p < .01)$.

***$t_{120} = 3.373$, $(p < .001)$; $t_{\infty} = 3.291$, $(p < .001)$.

predictors of success in math were also significant between treatments. The difference in this analysis was found in the interest and personality variables, which tended to interact between treatments (see Figures 2-18). All of the significant differences favored the geometric stars. This may be due to the fact that geometry grades were more reliable than algebra grades, or it may be due to intelligence levels of the students (i.e., geometric stars' mean IQ score was significantly greater than the algebraic stars' mean IQ score).

At the 0.001 level, the ITED verbal measures (Vocabulary, Uses of Information, Literature, Social Science Reading, Correctness of Expression, Natural Science Reading, Social Concepts) and the ITED Quantitative Thinking subtest produced significant differences as did IQ and the MAT Vocabulary and General Information subtests. Significant differences at $\alpha = 0.01$ and 0.05 were found on the ITED Natural Science Background; father's education level; PST Equation; vocal musical experience; EPI-IW factor; and SDII interest in geometry.

Although not significant at the 0.05 level, algebra tended to interact with interest in math, algebra and reading, whereas geometry grades interacted significantly with interest in geometry. Algebra also interacted with the "intellectual-nonconformist" personality. Geometry, on the other hand, interacted with the personality types, "intellectual-withdrawn" and "organizer-perfectionist." Students whose fathers had a high education level tended to have had some interaction with geometry. The geometric stars had a higher IQ score which may be the reason why IQ was such a high predictor of success in geometry and not algebra.

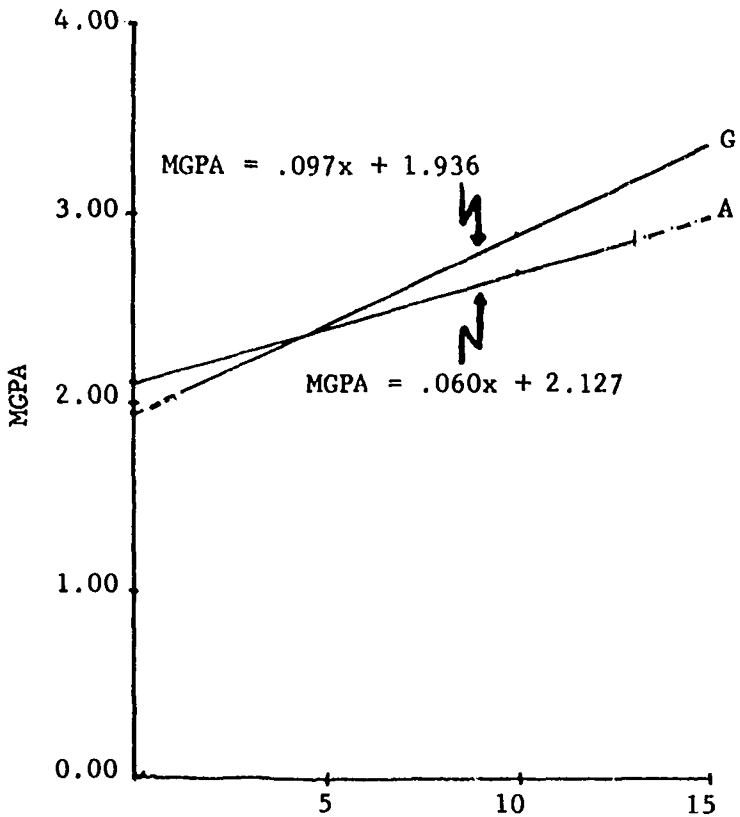


Figure 2. Interaction of MVOC with algebra (A) and geometry (G)

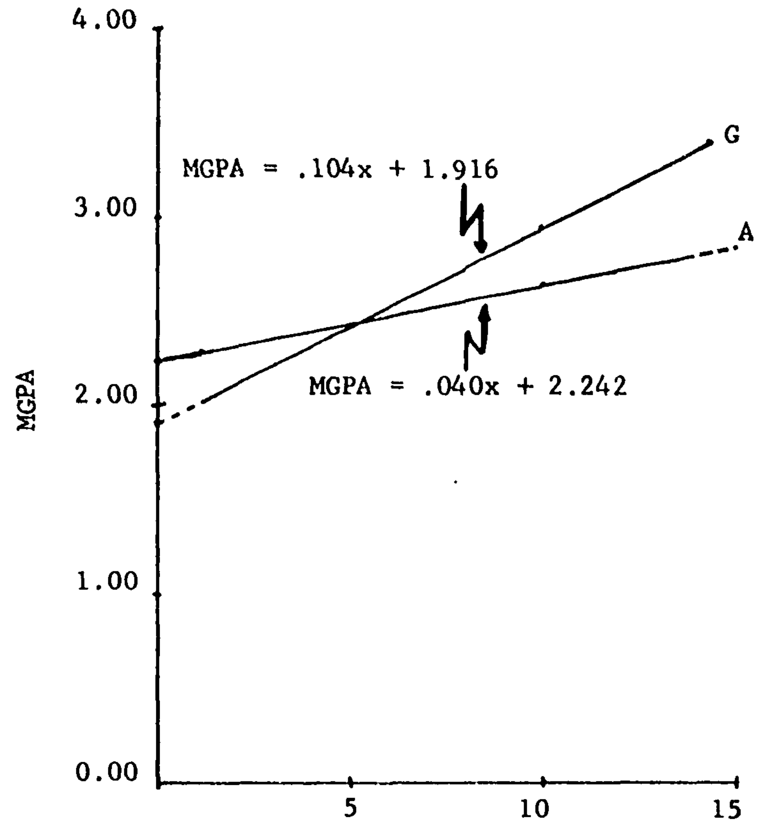


Figure 3. Interaction of MGENFO with algebra (A) and geometry (G)

Figures 2 to 18 show plots of separate regression lines for algebraic stars (n = 98) and for geometric stars (n = 124).

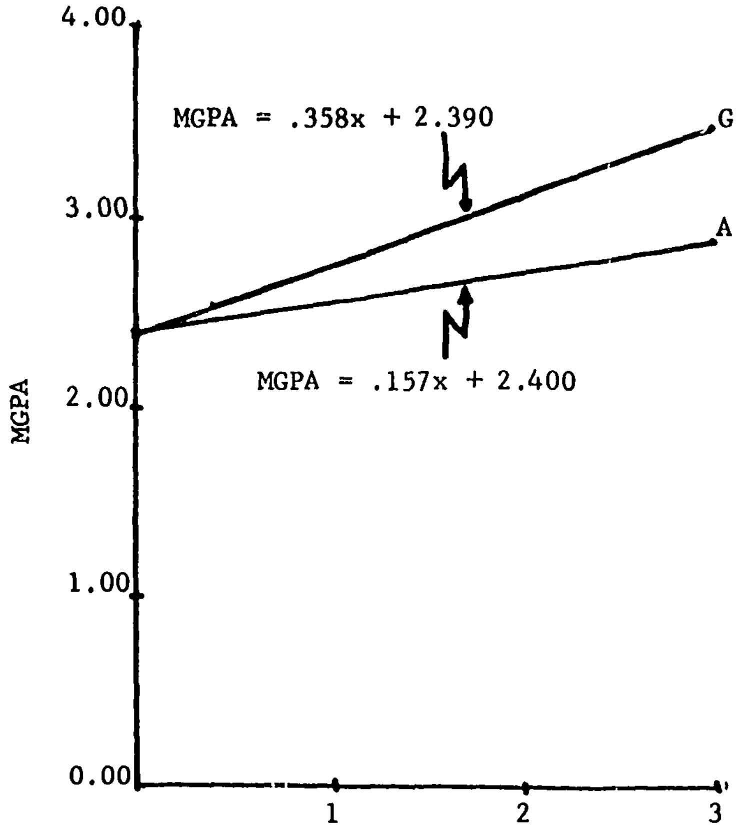


Figure 4. Interaction of EQUA with algebra (A) and geometry (G)

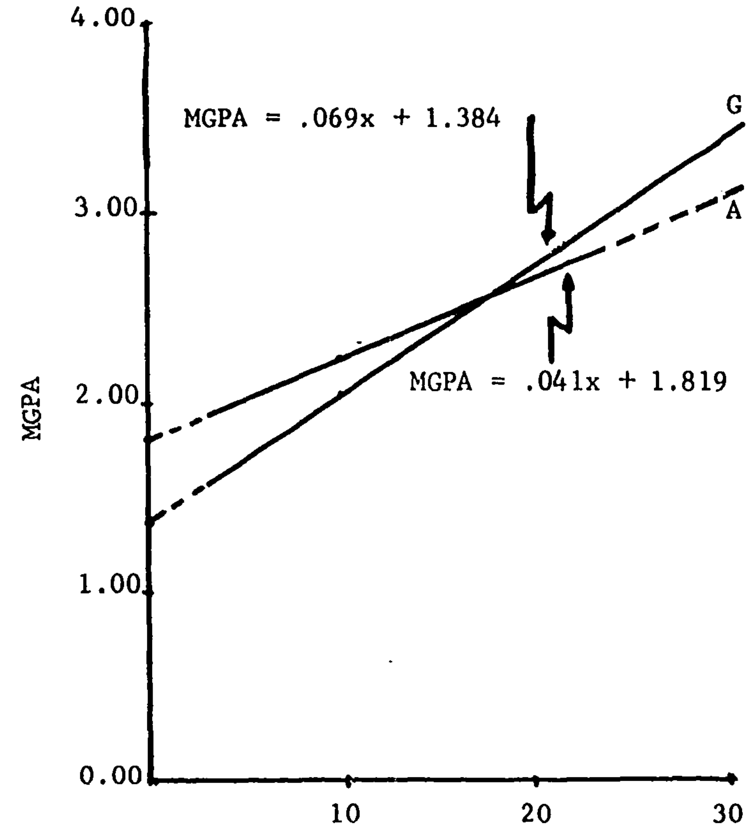


Figure 5. Interaction of SCON with algebra (A) and geometry (G)

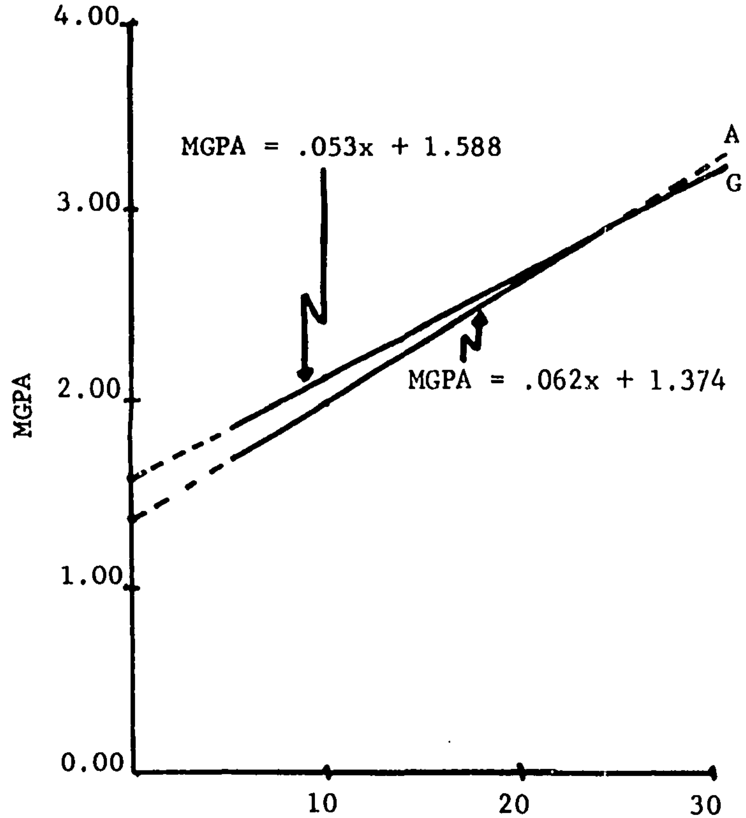


Figure 6. Interaction of NSBK with algebra (A) and geometry (G)

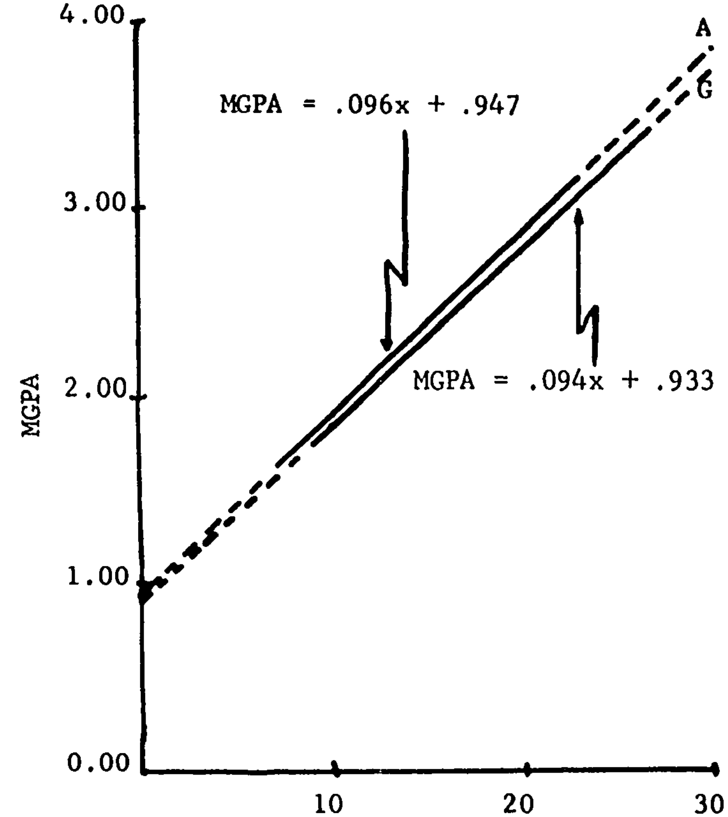


Figure 7. Interaction of EXPR with algebra (A) and geometry (G)

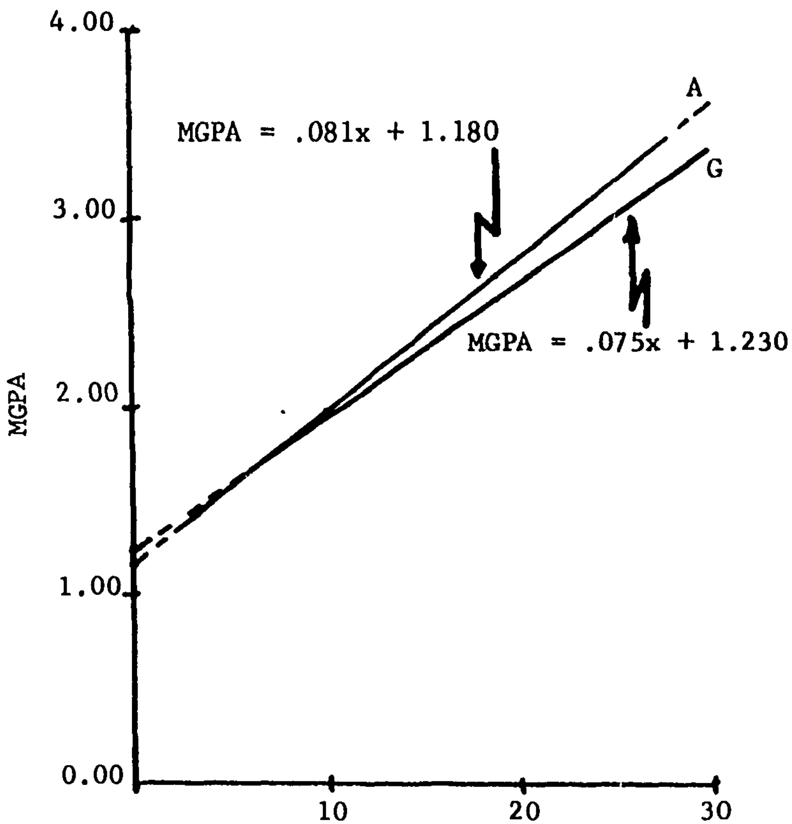


Figure 8. Interaction of QUANT with algebra (A) and geometry (G)

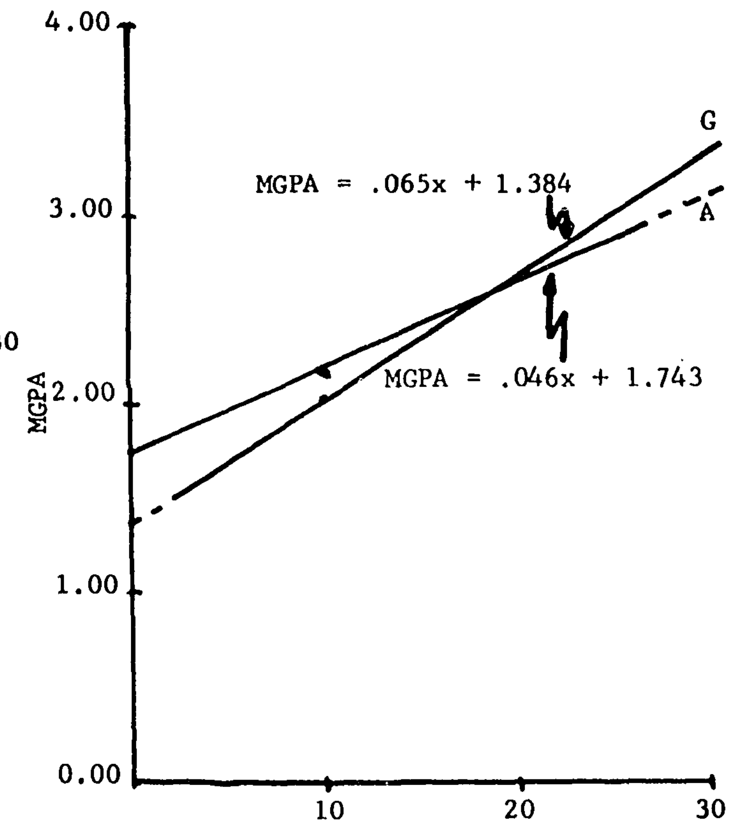


Figure 9. Interaction of SSREAD with algebra (A) and geometry (G)

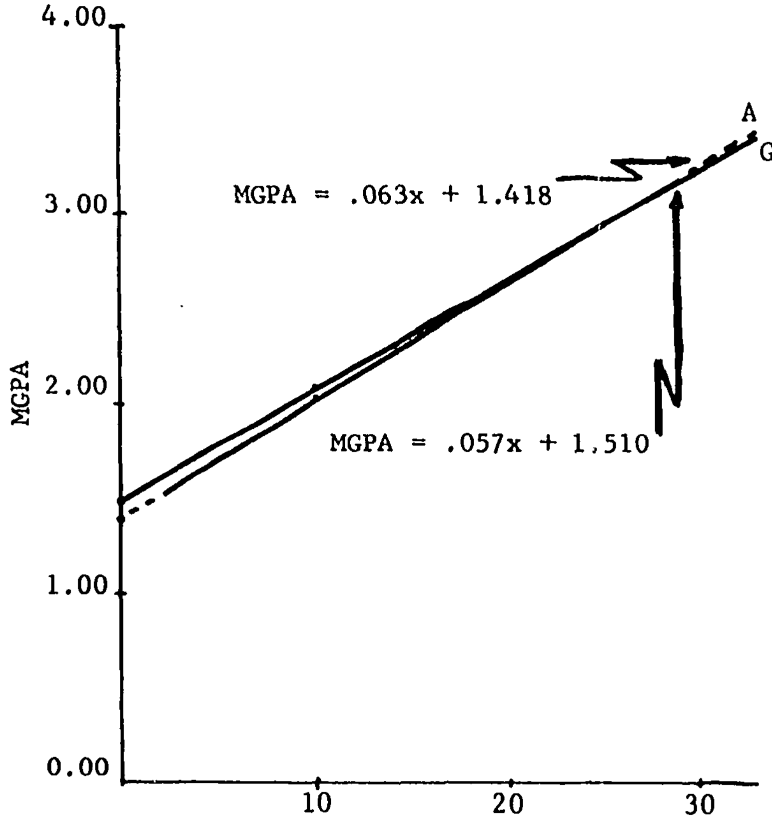


Figure 10. Interaction of NSREAD with algebra (A) and geometry (G)

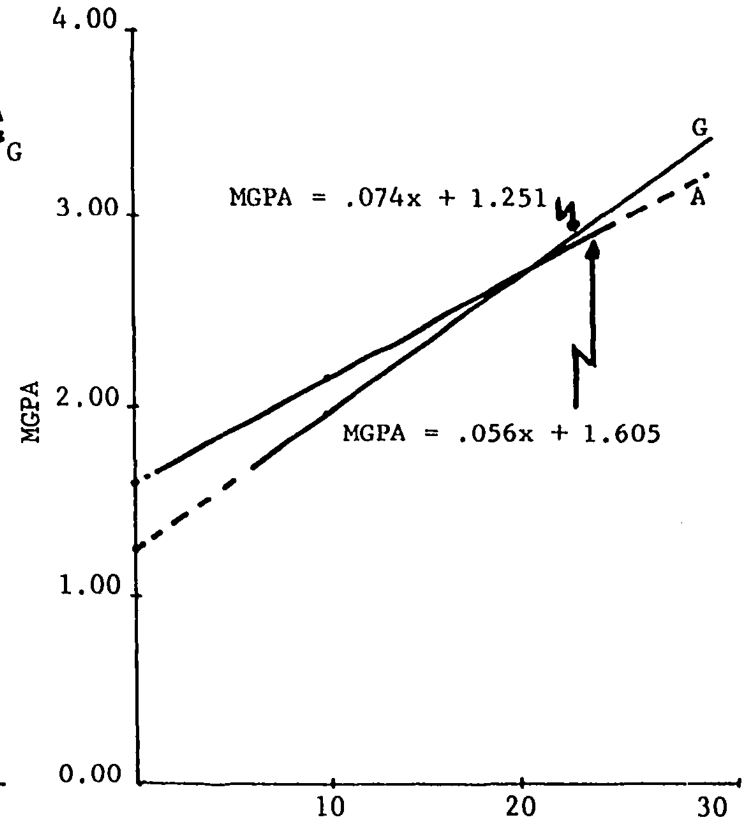


Figure 11. Interaction of LIT with algebra (A) and geometry (G)

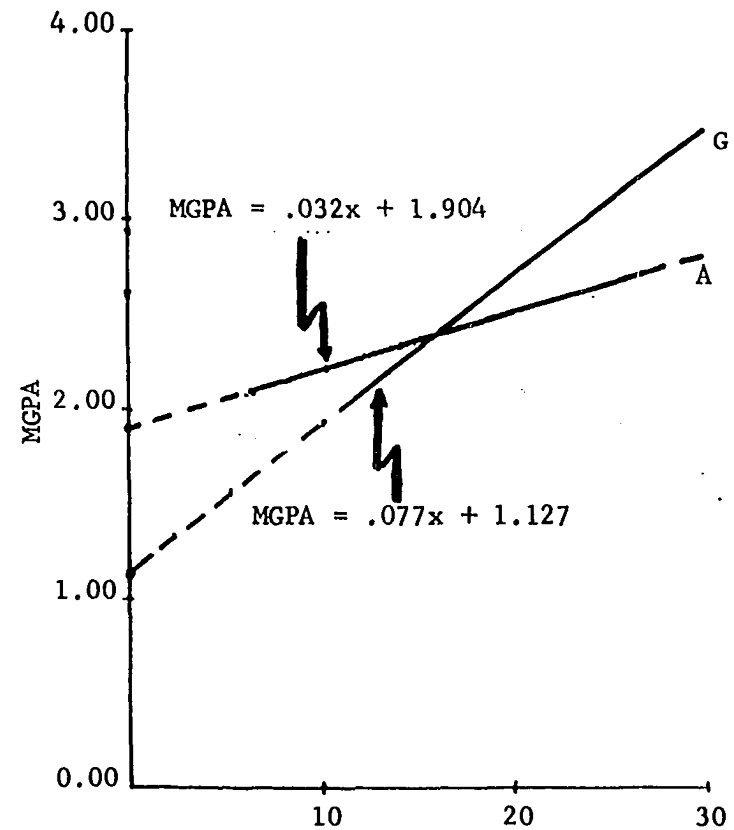


Figure 12. Interaction of VOCAB with algebra (A) and geometry (G)

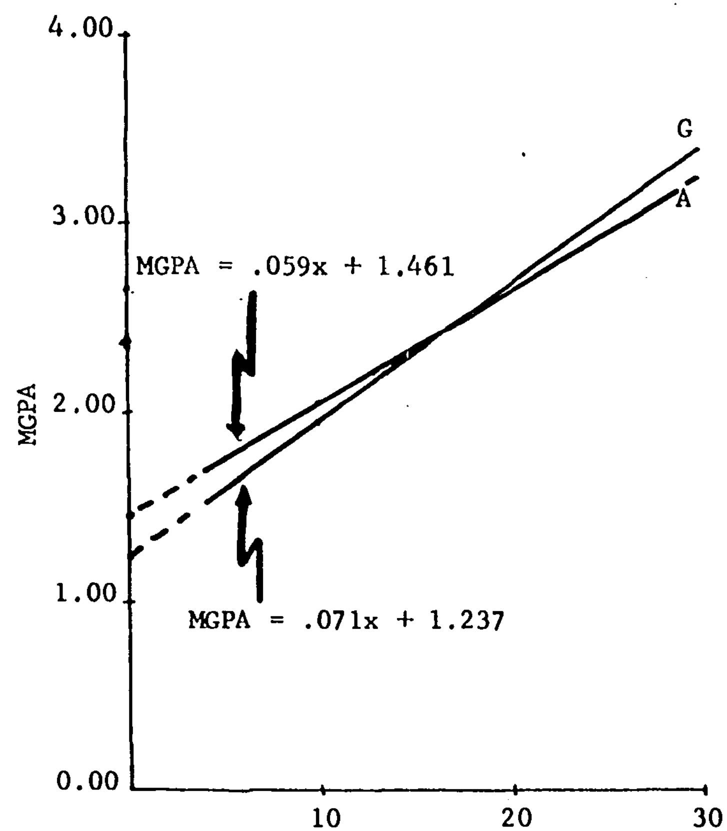


Figure 13. Interaction of USES with algebra (A) and geometry (G)

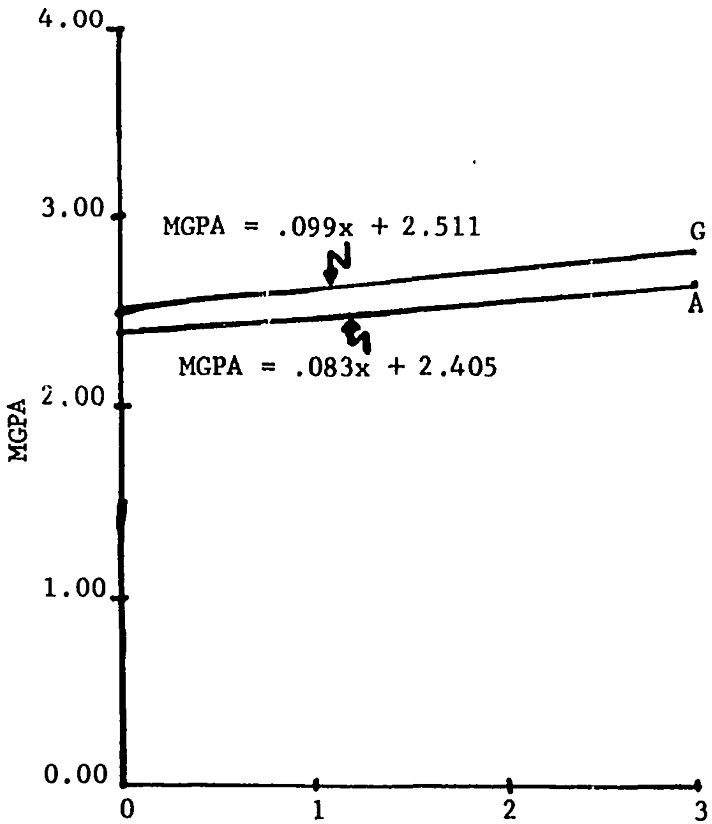


Figure 14. Interaction of <u>VOCAL</u> with algebra (A) and geometry (G)

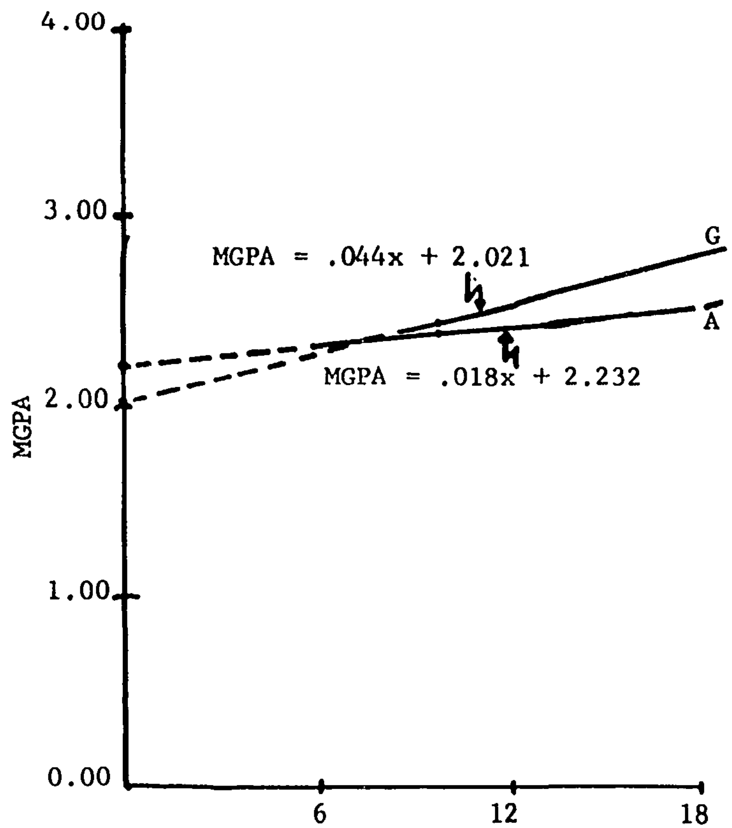


Figure 15. Interaction of <u>FED</u> with algebra (A) and geometry (G)

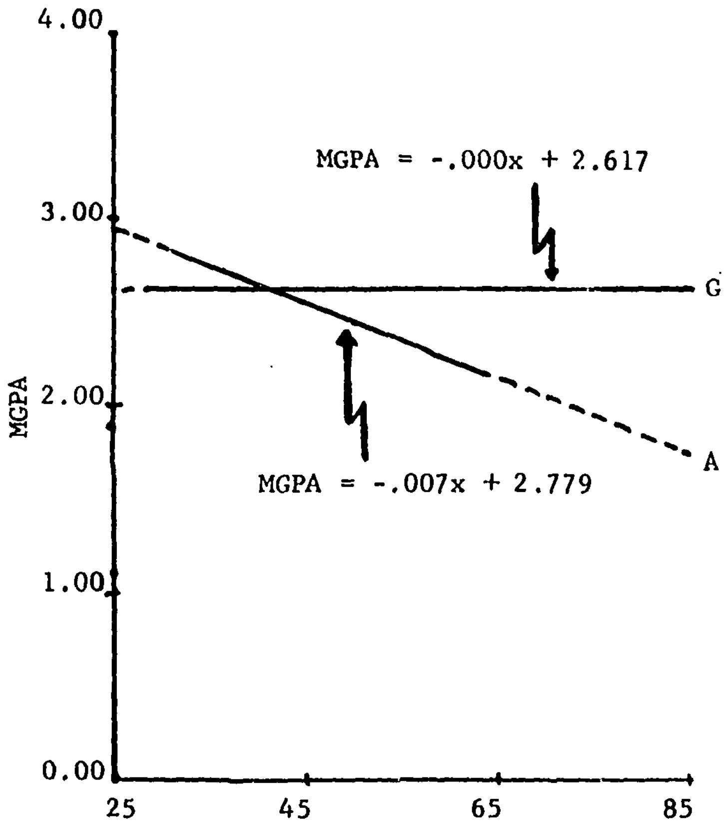


Figure 16. Interaction of EPI-IW with algebra (A) and geometry (G)

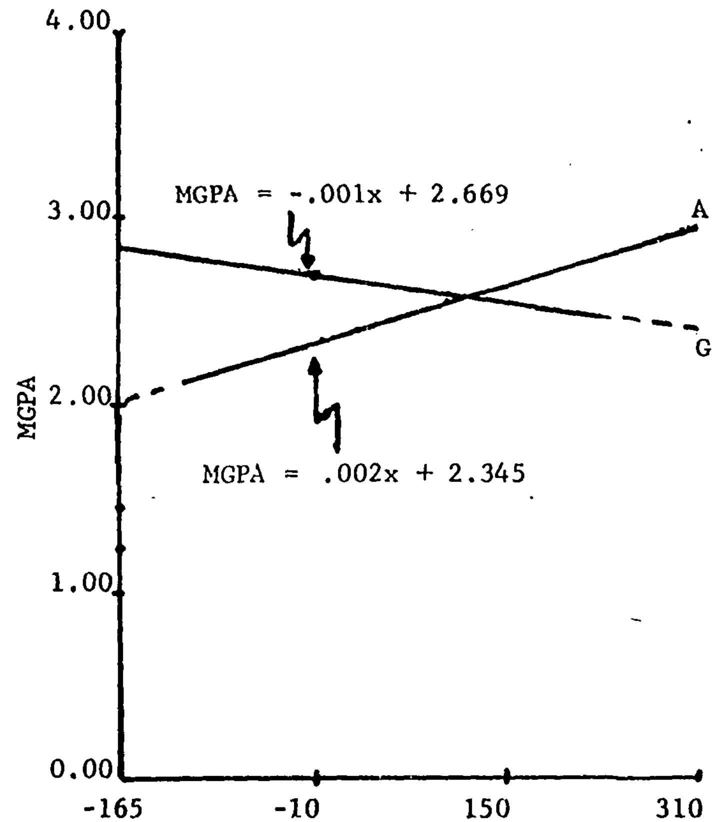


Figure 17. Interaction of SDII-G with algebra (A) and geometry (G)

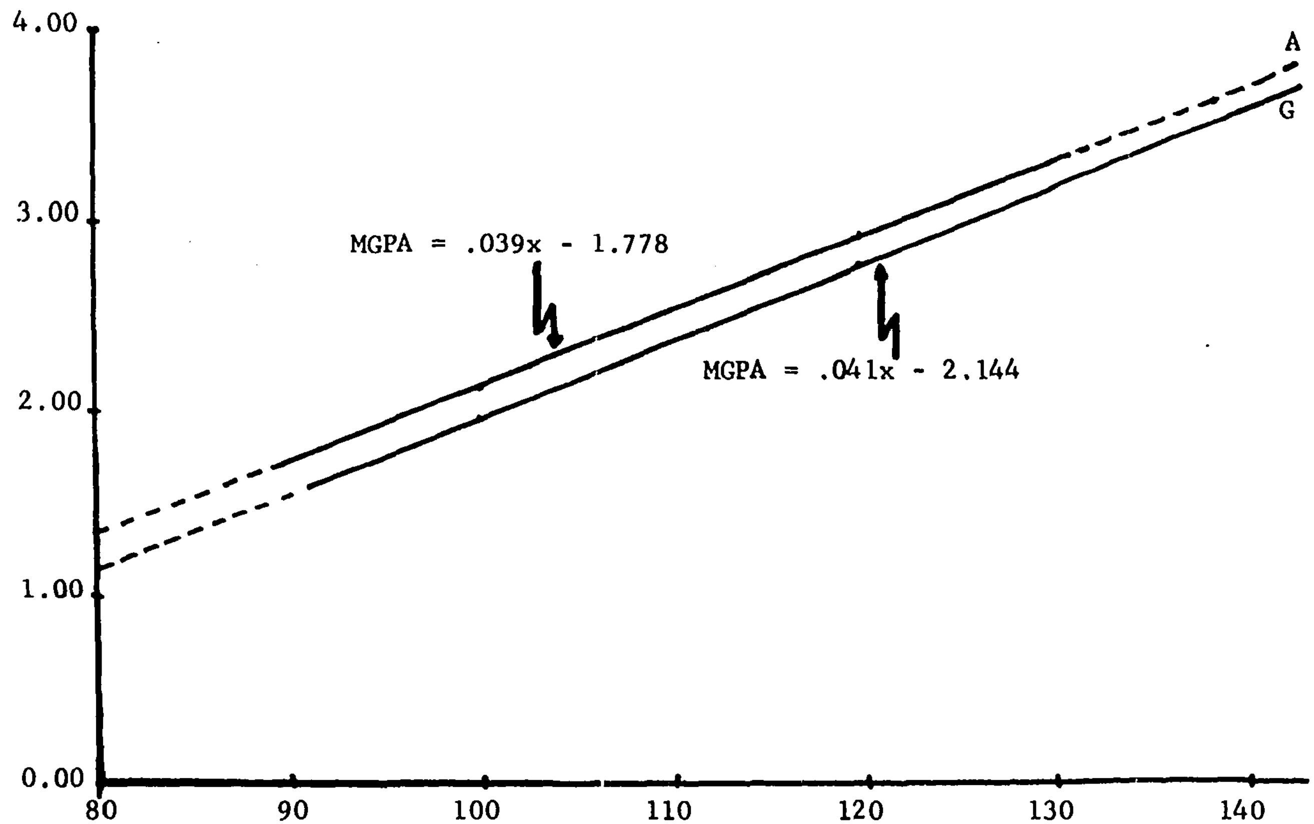


Figure 18. Interaction of IQ with algebra (A) and geometry (G)

Spatial perception did not interact with either course, but general reasoning had a high interaction with geometry. Geometry grades also interacted with verbal measures. The difference between algebraic and geometric stars was less on quantitative measures than on these verbal measures.

Summary

There were no significant differences between stars and nonstars on measures of personality, interest, sex, socioeconomic status, and music. Success in math was indicated by the interrelationship of these variables with achievement tests and tests of problem solving ability. Mathematical aptitude factors were found to be: quantitative thinking, arithmetic skills, general intelligence and reasoning, verbal ability, spatial perception, natural science reading ability and knowledge, and algebraic skills. Those factors which interacted with algebra were not as significant as those which interacted with geometry. Each subject appeals to different personality-types. Verbal ability and quantitative thinking interacts highly with geometry and the differences in computational ability of the two groups are insignificant. Interest in geometry was important to students' success in geometry; in contrast, interest in algebra was not significantly different between treatment groups. These results show that various combinations of personological variables indicate overall success in mathematics and that these factors interact differently with the treatments, algebra and geometry.

DISCUSSION AND CONCLUSIONS

Psychological traits and abilities can be measured by a wide variety of instruments and methods. However, even when two instruments purporting to measure the same ability are administered to the same sample a less than perfect correlation often results. Therefore, the findings of this study may differ from those of other studies due to variations in instruments used. Also, differences between samples of students studied may cause differences in results.

Correlations Between Variables

In this study, the intercorrelations between the 43 variables examined generally fell in the range of values found in previous studies. Correlations of IQ with other variables ranged from 0.13 to 0.45 for the ITED subtests; from 0.18 to 0.38 for the PST; and were essentially zero with socioeconomic status, personality and interest measures. These results are similar to those reported in the studies surveyed by Suydam (1970). IQ correlated 0.53 and 0.57 with geometry semester grades, which is comparable to Hummer's (1936) correlation of 0.58 between IQ and geometry test scores. Algebra grades, however, correlated lower with IQ than in Rosilda's (1951) study--0.32 and 0.34 as compared to 0.42. The three personality measures correlated higher with achievement test scores than with grades, interest measures, socioeconomic status or musical experience; results similar to those of Ayers, Bashaw and Wash (1969). Like Posamentier's findings (1966), verbal measures (on the ITED and MAT) correlated positively with geometry grades. These measures also correlated more highly with geometry grades than with algebra grades.

Achievement in math courses, as measured by semester grades, was found to be independent of student's interest in the subject matter. This result is opposed to the high positive correlations found in studies by Aiken (1972a), Ellingson (1962), and Schneider (1969). Since there may have been a restricted range of students within this study (i.e., all were drawn from one school system and all had two years of high school mathematics), interest in these subjects may not vary as widely as in the population at large. Socioeconomic status, as measured by parents' education levels and occupations, also had no relationship with grades except for a slight positive correlation between father's and mother's education levels and geometry grades. Unlike Anastasi's (1966) survey, the correlation matrix showed that the socioeconomic variables had near zero correlation with IQ. Math grades and sex correlated low, from 0.06 to 0.11, and slightly favored the girls.

As teacher ratings of students' mathematical aptitudes were made after the students' grades were assigned, these measures may not be independent, as shown by the positive correlations between teacher ratings and algebra semester grades (0.48 and 0.55) and geometry semester grades (0.66 and 0.67).

Indicators of Success

Formerly, success in mathematics was thought to be reserved for only the intelligent (Wrigley, 1958). In later investigations, however, other variables were found to be indicative of success in mathematics. High verbal ability (Muscio, 1962), personality factors (Cattell & Butcher, 1968), reading ability (Call & Wiggin, 1966), musical ability (Jenkins, 1961), interest in math (Aiken, 1972a), and socioeconomic data (Anastasi, 1966) have all been shown to predict or correlate with math achievement. But prior to this study, no investigation of the relative importance of each of these variables to success in mathematics has been carried out.

Those cognitive variables which predicted success in math were verbal ability (interpretation of natural science reading materials), quantitative ability (computational skills, quantitative thinking), and spatial perception. Affective measures having a discriminating role were: interest in math, feeling math is easy, and personality factor, EPI-OP, (the student is persistent, a perfectionist who likes to plan and organize things).

Prediction of algebra and geometry grades was similar to the prediction of overall success in math. However, some variables were better indicators of success in algebra than for success in geometry: interest in geometry and the personality factor, EPI-IW, (the student is intellectually oriented but does not like to venture into new situations). General intelligence and the ability to think logically were better predictors of success in geometry. IQ appears to be more closely related to geometry grades than algebra grades since students with the highest IQs were geometric stars.

As students within this sample have already experienced some degree of success in math, evidenced by their electing to study both algebra and geometry, and the fact that only students who completed both courses were included in the study, the range in scores on variables related to math ability is perhaps narrower than it would be for the entire high school student body. Consequently, correlations between predictors and math grades may have been lowered, thus resulting in some potentially useful predictors being statistically insignificant in this sample.

Factors of Mathematical Aptitude

Even within this restricted sample of students, it was possible to find variables which discriminated between high and low math achievers. Those factors which discriminated between high and low achievers and thus can be considered factors of mathematical aptitude are: quantitative thinking, computational skills, interpretation of natural science reading materials, vocabulary, use of sources of information, general reasoning and logical thinking, knowledge of science, interpretation of social science concepts, and algebraic skills. It is important to note that these factors of mathematical aptitude are to be taken collectively as a composite; it is the interdependence of these variables which indicate success, not a single high score on one of these measures. Thus, for example, to conclude that a student's failure in algebra was due to his poor algebraic skills would ignore the interrelationship of this skill to his reading ability or quantitative thinking ability.

When combining data from several instruments, as in this case, the measurement error in each variable reduces the amount of true score variance. In addition some factor analyses yielded incomplete solutions and thereby introducing possible error. Administrative conditions at one school were not ideal and may also have been a source of error.

The low percent of variance accounted for ($R^2 = .54$) may be attributable to these sources of error. Or, perhaps, other variables, such as science grades or music ability, need to be included in the regression in order to account for more of the variance in semester grades.

ATI

Mean differences between treatments were significant for 17 variables; all of these means were higher for the geometric stars. Generally, aptitude-treatment interactions followed the pattern, "high aptitude interacts favorably with a geometric treatment and low aptitude interacts with an algebraic treatment." As in most cases, the range of scores for algebraic and geometric stars did not overlap at the extremes, when examining the regression lines between the two treatments some aptitudes interacted more favorably with an algebraic treatment than with a geometric treatment.

Inspection of the projected regression lines led to the following conclusions: Students with high quantitative thinking ability did well with an algebraic treatment and students with low quantitative ability did better with a geometric treatment. Similar interactions favoring the algebraic treatment were observed on measures of natural science background, correctness of expression, IQ, and interest in geometry. Interest in geometry increased directly as math GPA increased for algebraic stars but for geometric stars interest in geometry was inversely related to math GPA.

Students with high verbal ability performed better with a geometric treatment and students who had lower verbal ability performed better on an algebraic treatment. Similarly, high scores on measures of social studies reading materials and social science concepts interacted with geometric treatment; low ability interacted with an algebraic treatment. Geometric treatments also produced interactions with students who were intellectually oriented, liked to be alone and did not seek new experiences. Geometric stars who had several years of vocal musical instruction and whose fathers had a high education level also experienced a high level of success in math. Students who demonstrated their ability to solve the system of simultaneous equations (PST-EQUA) performed better on a geometric treatment.

Implications for Education

Educators need to be aware of individual differences and how these differences affect and are affected by learning processes. The teacher who tries to individualize instruction and assignments recognizes the existence of individual differences but usually charts unknown areas when attempting to find suitable alternative treatments for students having varying abilities and personality characteristics. From aptitude-treatment interaction studies one may gain insights as to how to adjust the curriculum to the needs and abilities of the individual student. Two possible alternatives seem to exist. Teachers can either attempt to provide additional help and thus build up areas in which the student is deficient, or the teacher may develop and use alternative treatments which capitalize on the skills possessed by the individual student. The former step may be too time consuming for a school situation; the latter, on the other hand, can be dispatched more quickly. In either case, school counselors need to provide the teacher with necessary background data and curriculum developers must make materials for alternative instructional treatments easily accessible to the classroom teacher.

Algebra and geometry are two branches of mathematics which can be considered alternative treatments. This study found different patterns of aptitude to be associated with these two treatments. Success in geometry was related to high verbal ability, vocal musical experience, general reasoning ability, and reading ability. Algebra interacted with computational skill, natural science background, general intelligence, and interest in geometry.

From these findings we could conclude that students whose aptitudes interact with a geometric treatment would perform better in a math course which was taught with emphasis more on verbal and logical discussions. Similarly, for students whose aptitudes interact with an algebraic treatment, success should increase with more emphasis on computational skills and quantitative thinking and less emphasis on verbal discussions.

To illustrate, PST-3 (Ant Wants the Food) is an example of a problem which could be solved by either algebraic or geometric methods. By computing the hypotenuse of the figure in two-dimensions and comparing this value to other possible routes, the student will arrive at the correct solution by using his quantitative thinking ability, computational skill, and spatial perception abilities. The same solution may be attained by a geometric procedure involving logical thinking and general intelligence as well as spatial perception. Using this latter approach the student must compare lengths of lines, recall that the diagonal of the rectangle will result in the shortest distance between two points, then choose the diagonal of the largest rectangle in order to arrive at the solution.

The results of ATI studies indicate areas in which more flexibility in teaching methods could be introduced in order to allow for individual differences. Teachers need to provide more opportunities, such as in the above problem, for students to use their varying math aptitudes. In order to develop situations which allow for individual differences requires that teachers know which variables interact with which treatments. Acceptance of students' different approaches to attain the same goal is necessary. As a result of this study, math teachers may be better able to find problems and create discussions which utilize those aptitudes which were found to interact with the treatments, algebra and geometry.

Recommendations for Further Study

Different abilities were found to interact with algebraic and geometric treatments and to indicate success in these subjects. These indicators of success need to be further tested in a variety of ways. A replication of this study in another school system would check the consistency of the findings. To test the predictive power of the variables measures need to be taken from a sample of students before they begin studying algebra and geometry; a comparison of the resultant grades with the predicted grades could then be made. Researchers should investigate other instruments, methods, and variables in order to attempt to increase the percent of variance accounted for.

At another level, curriculum guides could be developed to help teachers integrate and adjust their lesson plans to the aptitudes of their students. A geometry guide might list those aptitudes which interact with a geometric approach to finding the total angle measurement of a triangle and suggest a lesson involving experimentation with a variety of cut-out triangles. Another, more algebraic, approach would involve an experiment with direct measurement of the angles of a triangle. Guides would provide a variety of test items which could be solved by either approach. The effectiveness of the various approaches--algebraic and geometric--to teaching the same concept could then be evaluated using the ATI paradigm.

BIBLIOGRAPHY

Ackerman, W., & Levin, H. Effects of training in alternative solution on subsequent problem solving. Journal of Educational Psychology, 1958, 49, 239-244.

Aiken, L. R., Jr. Attitudes toward mathematics. Review of Educational Research, 1970, 40, 551-596.

Aiken, L. R., Jr. Verbal factors and mathematics learning: A review of research. Journal for Research in Mathematics Education, 1971, 2, 304-313.

Aiken, L. R., Jr. Biodata correlates of math attitudes. School Science and Mathematics, 1972, 72, 386-395. (a)

Aiken, L. R., Jr. Research on attitudes toward mathematics. Arithmetic Teacher, 1972, 19, 229-234. (b)

Aiken, L. R., Jr. Ability and creativity in mathematics. (Unpublished manuscript) Guilford College, Greenboro, N.C. 1973.

Aiken, L. R., Jr. & Dreger, R. M. The effect of attitudes on performance in mathematics. Journal of Educational Psychology, 1961, 52, 19-24.

Alexander, V. E. Sex differences in seventh grade problem solving. School Science and Mathematics, 1962, 62, 47-50.

Alpert, R., Stellwagon, G., & Becker, D. Psychological factors in mathematics education. Report summary in Newsletter No. 15. School Mathematics Study Group, Stanford University, 1963.

Anastasi, A. Differential psychology: Individual and group differences in behavior. New York: Macmillan, 1958, vol. 3.

Anastasi, A. Individual differences. New York: Wiley, 1966.

Anttonen, R. G. A longitudinal study in mathematics attitude. Journal of Education Research, 1967, 62, 467-471. (ERIC EJ 007 271).

Atkinson, R. C., & Fletcher, J. D. Teaching children to read with a computer. Reading Teacher, 1972, 25, 319-327.

Ayers, J. B., Bashaw, W. L., & Wash, J. A. A study of the validity of the Sixteen Personality Factor Questionnaire in predicting high school academic achievement. Educational and Psychological Measurement, 1969, 29, 479-484. (ERIC EJ 005 062).

Backman, M. E. Patterns of mental abilities: Ethnic, socioeconomic, and sex differences. American Educational Research Journal, 1972, 9, 1-11.

Baldauf, R. J. Predicting success in eighth grade algebra. Psychological Reports, 1963, 12, 810.

Barnes, W. E., & Asher, J. W. Predicting students' success in first-year algebra. Mathematics Teacher, 1962, 55, 651-654.

Barton, K., Dielman, T. E., & Cattell, R. B. Personality and IQ measures as predictors of school achievement. Journal of Educational Psychology, 1972, 63, 398-404.

Becker, J. P. Research in mathematics education: The role of theory and of aptitude-treatment-interaction. Journal for Research in Mathematics Education, 1970, 1, 19-28.

Bennett, G. K., Seashore, H. G., & Wesman, A. G. A manual for the Differential Aptitude Tests. New York: The Psychological Corp., 1952.

Bingham, A. Improving children's facility in problem solving. New York: Columbia University, 1958.

Bingham, W. V. Aptitudes and aptitude testing. New York: Harper and Brothers, 1937.

Bloom, B. S., Hastings, J. T., & Madaus, G. F. (Eds.). Handbook on formative and summative evaluation of student learning. New York: McGraw-Hill, 1971.

Bond, E. A. Tenth-grade abilities and achievement. Contributions to Education, 1940, 813, 1-64.

Bowman, H. L. The relation of reported preference to performance in problem solving. The University of Missouri Bulletin, 1929, 30, 1-52.

Bowman, H. L. The relation of reported preference to performance in problem solving. Journal of Educational Psychology, 1932, 23, 266-276.

Bracht, G. H. The relationship of treatment tasks, personological variables, and dependent variables to aptitude-treatment interactions. Dissertation Abstracts, 1969, 30A, 4268. No. 70-5820.

Bracht, G. H. Experimental factors related to aptitude-treatment interactions. Review of Educational Research, 1970, 40, 627-646.

Brown, F. G. Principles of educational and psychological testing. Hinsdale: Dryden Press, 1970.

Burgert, R. H. The relation of school marks to intelligence in secondary schools. Journal of Applied Psychology, 1935, 19, 606-614

Cain, R. W. Relationships of verbal reasoning and numerical ability to achievement in first year algebra. School Science and Mathematics, 1966, 66, 131-134.

Call, R. J., & Wiggin, N. A. Reading and mathematics. Mathematics Teacher, 1966, 59, 149-157.

Callister, S. L. Stress, anxiety, and achievement relationships in programmed and conventional algebra and geometry classes. Dissertation Abstracts, 1965, 26, 5860. No. 66-4364.

Carey, G. L. Sex differences in problem-solving performance as a function of attitude differences. Journal of Abnormal and Social Psychology, 1958, 56, 256-260.

Carroll, J. B. A review of the Differential Aptitude Test. In O. K. Buros (Ed.), The Fifth Mental Measurements Yearbook. Highland Park, New Jersey: The Gryphon Press, 1959.

Cattell, R. B., & Butcher, H. J. The prediction of achievement and creativity. New York: Bobbs-Merrill, 1968.

Clark, M. I. The effect of remedial work in reading comprehension upon algebraic achievement. Mathematics Teacher, 1939, 32, 65-67.

Copeland, R. W. How children learn mathematics. New York: Macmillan, 1970.

Court, N. A. Mathematics in fun and earnest. New York: Mentor Books, 1958.

Covington, M. V. A childhood attitude inventory for problem solving. Journal of Educational Measurement, 1966, 3, 234.

Coy, G. L. Interests, abilities, and achievements of gifted children. Contributions to Education, 1923, 131, 16-28.

Cronbach, L. J. The two disciplines of scientific psychology. Address of the president at the meeting of the American Psychological Association, New York, September 1957.

Cronbach, L. J. How can instruction be adapted to individual differences? In R. M. Gagné (Ed.), Learning and individual differences. Columbus, Ohio: Merrill Books, 1967.

Cronbach, L. J., & Gleser, G. C. Psychological tests and personnel decisions. Urbana: University of Illinois Press, 1965.

Cronbach, L. J., & Snow, R. E. Individual differences in learning ability as a function of instructional variables. Stanford: Stanford University, School of Education, 1969.

Crow, L. D., & Crow, A. Human development and learning. New York: American Book Company, 1965.

Cureton, E. E., Cureton, L. W., et al. The Multi-Aptitude Test. New York: Psychological Corp. 1955.

Curry, J. F. The effect of reading instruction upon achievement in seventh-grade arithmetic. Dissertation Abstracts, 1955, 15, 2059. No. 14-572.

D'Augustine, C. H. Factors relating to achievement with selected topics in geometry and topology when taught to fifth-, sixth- and seventh-grade pupils via a programed text. Dissertation Abstracts, 1963, 24, 4538-4539. No. 64-3586.

Daniels, S. The value of not following the directions. Instructor, 1971, 81, 21.

Davis, F. W. A study of three methods of utilizing programed algebra textbooks. Dissertation Abstracts, 1966, 27A, 2272-2273. No. 66-15,300.

de Bono, E. New think. New York: Basic Books, 1968.

Degnan, J. A. General anxiety and attitudes toward mathematics in achievers and underachievers in mathematics. Graduate Research in Education and Related Disciplines, 1967, 3, 49-62.

Dinkel, R. E. Prognosis for studying algebra. Arithmetic Teacher, 1959, 6, 317-319.

Dresher, R. Training in mathematics vocabulary. Educational Research Bulletin, 1934, 13, 201-204.

Dunlap, J. W. Race differences in the organization of numerical and verbal abilities. Archives of Psychology, 1931, 19, 1-17.

Dwyer, F. M., Jr. Visual learning: An analysis by sex and grade level. California Journal of Educational Research, 1971, 22, 170-176.

Eagle, E. The relationship of certain reading abilities to success in mathematics. Mathematics Teacher, 1948, 41, 175-179.

Ellingson, J. B. Evaluation of attitudes of high school students toward mathematics. Dissertation Abstracts, 1962, 23, 1604. No. 62-4946.

Farley, Sister M. de Chantal. A study of mathematical interests, attitudes, and achievements of tenth and eleventh grade students. Dissertation Abstracts, 1969, 29A, 3039-3040. No. 69-2312.

Ferderbar, J. E. Changes in selected student attitudes and personality measures and their relationship to achievement, intelligence, and rate when using programmed instruction. Dissertation Abstracts, 1965, 25, 6310. No. 65-5202.

Filter, R. O., & Held, O. C. The growth of ability. York, Pa.: The Maple Press Company, 1930.

Finck, E. M. Relation of ability in reading to success in other subjects. Elementary School Journal, 1935, 36, 260-267.

Foran, T. G., & O'Hara, Brother Colombiere. Sex differences in achievement in high-school geometry. School Review, 1935, 43, 357-362.

Frederiksen, N. A review of the Differential Aptitude Test. In O. K. Buros (Ed.), The Fifth Mental Measurements Yearbook. Highland Park, New Jersey: The Gryphon Press, 1959.

French, J. W. Effect of anxiety on verbal and mathematical examination scores. Educational and Psychological Measurement, 1962, 22, 553-564.

Gardner, M. The second scientific American book of mathematical puzzles and diversions. New York: Simon and Schuster, 1961.

Garrett, C. S. The Problem-solving skills associated with sex, academic major, and grade-point average. Unpublished M.S. thesis, Ames, Iowa, Library, Iowa State University of Science and Technology, 1970.

Gawronski, J. An informal approach to high school geometry. School Science and Mathematics, 1972, 72, 281-283.

Gilmary, Sister M. Transfer effects of reading remediation to arithmetic computation when intelligence is controlled and all other school factors are eliminated. Arithmetic Teacher, 1967, 14, 17-20.

Glaser, R. Individuals and learning: The new aptitudes. Educational Researcher, 1972, 1, 5-13.

Goldberg, L. R. A review of the Edwards Personality Inventory. In O. K. Buros (Ed.), The Seventh Mental Measurements Yearbook, vol. 1. Highland Park, New Jersey: The Gryphon Press, 1972.

Goldberg, L. R., & Grobman, H. Helping pupils fail algebra. Clearing House, 1972, 46, 468-470.

Haggard, E. A. Socialization, personality, and academic achievement in gifted children. School Review, 1957, 65, 388-414.

Hanna, G. S. An attempt to validate standard Kuder scales and an empirically-derived interest scale for predicting success in high school geometry. Educational and Psychological Measurement, 1966, 26, 445-448.

Hanna, G. S. The use of students' predictions of success in geometry and year of high school to augment predictions made from test scores and past grades. Journal of Educational Measurement, 1967, 4, 137-141.

Hanna, G. S., & Lenke, J. M. Moderator variables in predicting geometry success. School Science and Mathematics, 1970, 70, 299-302.

Hedley, C. N. The relationship of personality factors to scientific and mathematical ability factors. School Science and Mathematics, 1968, 68, 265-271.

Horst, P. The prediction of personal adjustment. Social Science Research Council Bulletin No. 48, 1941, 1-455.

Hudgins, B. B. Problem solving in the classroom. New York: Macmillan, 1967.

Hummer, V. L. A comparison of IQ and achievement in plane geometry. School Science and Mathematics, 1936, 36, 496-501.

Husén, T. (Ed.) International study of achievement in mathematics. New York: Wiley, 1967.

Jacobs, J. N. A study of performance of slow learners in the Cincinnati Public Schools on mental and achievement tests. American Journal of Mental Deficiences, 1957, 62, 238-243.

Jenkins, T. V. A study of the relationship between music aptitudes and mental ability, science aptitudes, and mathematics aptitudes among secondary school pupils in Texas. Dissertation Abstracts, 1961, 21, 2592. No. 60-6625.

Johnson, R. M. A comparison of gifted adolescents from high and low socioeconomic backgrounds on school achievement and personality traits. Dissertation Abstracts, 1966, 27A, 3226. No. 67-3945.

Joseph, M. The factor of interest in the teaching of mathematics. School Science and Mathematics, 1940, 40, 201-207.

Kauffman, M. M. Expressed interests of children in relation to a maturity-age index in grades four through eight. Dissertation Abstracts, 1955, 15, 2074. No. 13-156.

Kieren, T. E. Activity learning. Review of Educational Research, 1969, 39, 509-522.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: An exploratory study. Dissertation Abstracts, 1967, 28A, 4380. No. 68-4380.

Kilpatrick, J. Problem solving in mathematics. Review of Educational Research, 1969, 39, 523-534.

Kleinmuntz, B. (Ed.) Problem solving: Research, method, and theory. New York: Wiley, 1966.

Kochler, L. H. Using computer assisted instruction in overcoming attitude barriers. Unpublished doctoral dissertation, Ames, Iowa, Library, Iowa State University, 1972.

Kochnower, W. Personality factors and success in mathematics. High Points, 1961, 43, 65-72.

Lemon, A. C. An experimental study of guidance and placement of freshmen in the lowest decile of the Iowa Qualifying Examination, 1925. Iowa City, Iowa: University of Iowa, 1927.

Lindgren, H. C., Silve, I., Faraco, I., & Da Rocha, N. S. Attitudes toward problem solving as a function of success in arithmetic in Brazilian elementary schools. Journal of Educational Research, 1964, 58, 44-45.

Litsky, M. S. Different approaches to the teaching of a math unit. Graduate Research in Education and Related Disciplines, 1968, 4, 54-65.

Lovell, K. R. Intellectual growth and understanding mathematics: Implications for teaching. Arithmetic Teacher, 1972, 19, 277-282.

Luchins, A. S. Mechanization in problem solving. Psychological Monographs, 1942, 54, 1-95. No. 248.

Mallory, V. S. The relative difficulty of certain topics in mathematics for slow-moving ninth grade pupils. Contributions to Education, 1939, 769, 1-180.

Martin, M. D. Reading comprehension, abstract verbal reasoning, and computation as factors in arithmetic problem solving. Dissertation Abstracts, 1963, 24, 4547. No. 64-3395.

McCallon, E. L., & Brown, J. D. A semantic differential instrument for measuring attitude toward mathematics. The Journal of Experimental Education, 1971, 39, 69-72.

McCuen, T. L. Predicting success in algebra. Journal of Educational Research, 1930, 21, 72-74.

McIntosh, J. A. (Ed.) Perspectives on secondary mathematics education. New Jersey: Prentice-Hall, 1971.

McQueen, R., & Williams, K. C. Predicting success in beginning high school algebra. Psychological Reports, 1958, 4, 603-606.

Messler, D. L. A study of pupil age and achievement in eighth-grade algebra. Mathematics Teacher, 1961, 54, 561-564.

Mogull, R. G., & Rosengarten, W., Jr. Predicting student success in elementary algebra. Journal of Educational Research, 1971, 65, 104-107.

Morgan, J. H. The relationship between levels of anxiety and mathematics achievement in programed and teacher-directed instruction. Dissertation Abstracts, 1965, 26, 3760. No. 66-877.

Morrison, D. F. Multivariate statistical methods. New York: McGraw-Hill, 1967.

Muscio, R. D. Factors related to quantitative understanding in sixth grade. Arithmetic Teacher, 1962, 9, 258-262.

Nie, N., Bent, D. H., & Hull, C. H. Statistical package for the social sciences. New York: McGraw-Hill, 1970.

Norman, W. T. Review of the Edwards Personality Inventory. In O. K. Buros (Ed.), The Seventh Mental Measurements Yearbook. Highland Park, New Jersey: The Gryphon Press, 1972.

Nunnally, J. C. Psychometric theory. New York: McGraw-Hill, 1967.

Oakland, J. A. Measurement of personality correlates of academic achievement in high school students. Journal of Counseling Psychology, 1969, 16, 452-457.

Osgood, C. E., Suci, G. J., & Tannenbaum, P. H. The measurement of meaning. Urbana: University of Illinois, 1957.

Pack, E. C. The effects of testing upon attitude towards the method and content of instruction. Journal of Educational Measurement, 1972, 9, 141-144.

Phillips, R. B., Jr. Teacher attitude as related to student attitude and achievement in elementary school mathematics. Dissertation Abstracts, 1969, 30A, 4316. No. 70-4823.

Polya, G. Mathematical discovery: On understanding, learning, and teaching problem solving. New York: Wiley, 1965, vol. 2.

Posamentier, A. The effect of reading ability on performance on the Orleans Geometry Prognosis Test. Graduate Research in Education and Related Disciplines, 1966, 2, 184.

PREP Putting Research into Educational Practice. Research on Elementary Mathematics. (PREP Report No. 11.) Washington, D.C.: U.S. Department, HEW, National Institute of Education, 1972.

Romberg, T. A. Current research in mathematics education. Review of Educational Research, 1969, 39, 473-491.

Rosilda, Sister M. Is IQ an index to algebraic ability? Journal of Education Research, 1951, 44, 391-393.

Salamon, G. Heuristic models for the generation of aptitude-treatment interaction hypotheses. Review of Educational Research, 1972, 42, 327-343.

Schneider, A. J. An investigation of the relationships between self-concept of ability, achievement, and level of occupational aspiration among ninth grade boys. Dissertation Abstracts, 1969, 30A, 3285. No. 70-2942.

Service, J. A user's guide to the statistical analysis system. Raleigh, North Carolina: North Carolina State University, 1972.

Shapiro, E. W. Attitudes toward arithmetic among public school children in the intermediate grades. Dissertation Abstracts, 1961, 22, 3927. No. 62-1222.

Shaw, M. E. Scaling group tasks: A method for dimensional analysis. (Contract NR-70-266, Nonr-580, 11) Gainesville, Florida: Office of Naval Research, 1963.

Sheehan, T. J. Patterns of sex differences in learning mathematical problem-solving. Journal of Experimental Education, 1968, 36, 84-87.

Snider, J. G., & Osgood, C. E. Part I: Origin and nature of the semantic differential technique. Semantic Differential Technique A Sourcebook. Chicago: Aldine Publishing Company, 1969.

Sorensen, P. Program for learning in accordance with needs. Education, 1971, 92, 110-112.

Stroud, J. B., & Lindquist, E. F. Sex differences in achievement in the elementary and secondary schools. Journal of Educational Psychology, 1942, 33, 657-667.

Suppes, J. The Stanford Institute on Teaching for Reflective Thinking. Director's Report, 1971. (ERIC ED 054 048).

Suppes, P., & Feldman, S. Young children's comprehension of logical connectives. Journal of Experimental Child Psychology, 1971, 12, 304-317.

Suydam, M. N. Annotated compilation of research on secondary school mathematics, 1930-1970. (Project No. 1-C-004). University Park, Penn.: U.S. Department, HEW, 1972, vol. 1, 2.

Tate, M. W., & Stanier, B. Errors in judgment of good and poor problem solvers. Journal of Experimental Education, 1964, 32, 371-376.

Terman, L. M., et al. Genetic studies of genius: Vol. I. mental and physical traits of a thousand gifted children. Stanford, Calif.: Stanford University Press, 1925.

Thorndike, R. L. (Ed.) Educational measurement. Washington, D.C.: American Council on Education, 1971.

U.S. Bureau of the Census. 1950 Census of Population: Classified index of occupations and industries. Washington, D.C.; Author, 1950.

Weschler, D. The range of human capacities. Baltimore: Waverly Press, 1935.

Werdelin, I. A. A synthesis of two factor analyses of problem solving in mathematics. Didakometary, No. 8. Malmö, Sweden: Malmö School of Education, 1966.

Werdelin, I. The value of external direction and individual discovery in learning situations. 1, the learning of a mathematical principle. 2, the learning of a foreign alphabet, Dikakometry, No. 57 and 58. Stockholm, Sweden: School of Education, 1968. (ERIC ED 029 541).

Williams, J. W., & Warf, S. L. Education directory 1972-1973 public school systems. Washington, D.C.: U.S. Department, HEW, Elementary and Secondary Office, 1973.

Wittrock, M. C., & Wiley, D. E. (Ed.) The evaluation of instruction. New York: Holt, Rinehart and Winston, 1970.

Wrigley, J. Factorial nature of ability in elementary mathematics. British Journal of Educational Psychology, 1958, 28, 61-78.

Zamboni, F. F. A study of the effect of two different classroom procedures upon student achievement, anxiety and attitudes of second year high school Algebra II/Trigonometry students. Dissertation Abstracts, 1968, 29B, 3004-3005. No. 69-2878.

ACKNOWLEDGMENTS

I would like to express my grateful appreciation to those people who were instrumental in this research project: Dr. Fred Brown, for his relentless guidance, insight, edits and application of his quantitative thinking ability; Dr. "Tony" Netusil, for the initial direction and continued support; Dr. Rex Thomas, for his concern and willingness to sacrifice his time; Drs. Dick Warren and Leroy Wolins, for sharing their expertise in statistical analysis; and Dr. Harold Dilts, for his comments on the project at various stages.

Thanks is extended to the students and faculty involved at the four schools who gave up their time so that data could be collected, and especially to Mr. Lyle Latimer (Coordinator of Special Services, Council Bluffs school district), Mr. Hart and Mrs. Mary Veline (counselors, Council Bluffs), Mr. Ken Kuester (principal, Council Bluffs), Mr. Dennis Wermersen (North Polk Sr. High math teacher), Mr. Tom Lorntson and Mr. Ed Miller (Gilbert Community school math teachers).

To my parents, I would like to express my appreciation for providing for my education and motivating me throughout the years.

To my friends, Joan and Barb, without whom little would have been accomplished, I wish to express my appreciation for their selflessness, encouragement, intellectual stimulation and assistance in data manipulation.

APPENDIX A.1: MULTI-APTITUDE TEST

THE MULTI-APTITUDE TEST

NAME______________________ DATE______

SCHOOL________________________________

GRADE__________ SEX______

GENERAL DIRECTIONS

This test consists of six parts measuring different aptitudes and abilities. Each part has its own time limit. The time limits are short. WORK ON EACH PART ONLY DURING THE TIME ALLOWED FOR IT. If you finish a part before time is called, go back and check your work on that part. DO NOT return to a previous part, or go ahead to a later part. Work RAPIDLY on each part, but try not to make mistakes.

Each part has its own special directions, and one or two examples, correctly marked. Be sure you understand the directions for each part BEFORE you start to work on it. The examiner will NOT answer any questions after the starting signal for a part has been given.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

I. VOCABULARY

Each test word, in capital letters, is followed by five possible answers. The correct answer is the word which *means most nearly the same* as the test word. Make a *heavy* line with your pencil between the pair of dotted lines at the right which are lettered the same as the correct answer. EXAMPLE:

				A	B	C	D	E
FREQUENT:	A) always	B) often		⁞	▌	⁞	⁞	⁞
C) never	D) very	E) soon						

"Often" means most nearly the same as "frequent," so a heavy line has been made between the dotted lines at the right under B.

Mark an answer for every word. If you don't know the meaning of a word, make the best choice you can.

You will have *three minutes* to work on this test.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. EXTRAVAGANT: A) exclusive B) prodigious C) truant D) covetous E) excessive

2. HOMAGE: A) fodder B) toll C) allegiance D) foolishness E) fervor

3. IMMERSE: A) suspend B) anoint C) disclose D) submerge E) originate

4. ALIENATE: A) impoverish B) estrange C) dissipate D) conciliate E) deprecate

5. GARNISH: A) wield B) harrow C) toughen D) beautify E) degrade

6. PRECARIOUS: A) intimate B) wary C) invaluable D) perilous E) adventurous

7. DIABOLIC: A) disrupting B) dictatorial C) demented D) fiendish E) angelic

8. SAVOUR: A) relish B) poise C) balm D) fragrance E) prudence

9. QUAIL: A) recoil B) stimulate C) rout D) whiten E) descry

10. IMBUE: A) distort B) refute C) abstain D) inoculate E) allege

11. AFFRONT: A) opulence B) admittance C) reversion D) deception E) indignity

12. ANTIPATHY: A) animosity B) discomfiture C) sobriety D) clemency E) negation

13. WILE: A) frontier B) stealth C) force D) verdure E) stratagem

14. LEVITY: A) assessment B) frivolity C) solemnity D) residue E) annihilation

15. DROLL: A) apprehensive B) obtuse C) pitiable D) ludicrous E) listless

STOP. GO BACK AND CHECK YOUR WORK.

II. GENERAL INFORMATION

Each problem consists of a question or an incomplete sentence, followed by four possible answers. Choose the answer which *best* answers the question or completes the sentence, and put a cross in the box at the right which is lettered the same as the answer you have chosen. EXAMPLE:

Sausage is ordinarily made from

A) beef	B) mutton	A	B	C	D
C) pork	D) venison	☐	☐	☒	☐

Sausage is ordinarily made from pork, so a cross has been put in the box at the right under C.

If you don't know the answer to a problem, make the best choice you can. Leave it blank only if you have no hunch whatever about any of the answers.

You will have *two minutes* to work on this test.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Scrooge appears in
A) Henry IV B) Vanity Fair
C) Canterbury Tales D) The Christmas Carol

2. Coral is found in
A) reefs B) mines C) oysters D) elephants

3. The Amazon lies chiefly in
A) Chile B) Brazil C) Bolivia D) Argentina

4. Which is a chemical element?
A) salt B) steel C) brass D) mercury

5. A fuse is used primarily for
A) speed B) safety C) economy D) efficiency

6. The most prominent industry of Chicago is
A) steel B) textiles C) packing D) automobiles

7. Bile is secreted by the
A) liver B) spleen C) kidneys D) pancreas

8. Which of these is *not* a trade name?
A) stetson B) frigidaire
C) scotch tape D) ginger ale

9. The rutabaga is a
A) fish B) tree C) lizard D) vegetable

10. Which countries are most closely associated with the development of the ballet?
A) France and England B) France and Russia
C) Germany and Russia D) England and the U.S.

11. Which was settled first?
A) Boston B) Santa Fe
C) Plymouth Rock D) St. Augustine

12. With what architectural form is the "flying buttress" associated?
A) Gothic B) Georgian
C) Egyptian D) Romanesque

13. The creel is used in
A) hunting B) fishing
C) dancing D) sculpturing

14. Frank Lloyd Wright is noted for his work in
A) politics B) aviation
C) sculpture D) architecture

15. What is the final stage in passing an amendment to the constitution?
A) state ratification B) presidential signature
C) congressional action D) supreme court decision

STOP. GO BACK AND CHECK YOUR WORK.

III. ARITHMETIC

Perform the indicated operations for each problem, and write the answer in the box provided for it. Use the margin for figuring whenever necessary. In all problems involving fractions, reduce your answers to MIXED NUMBERS, with the fractional parts in their LOWEST TERMS. EXAMPLE:

$$\begin{array}{r} 1\frac{3}{4} \\ +\ 2\frac{3}{4} \\ \hline \boxed{4\frac{1}{2}} \end{array}$$

The answer MUST be $4\frac{1}{2}$, NOT $4\frac{2}{4}$ or $\frac{18}{4}$ or $\frac{9}{2}$.

You will have *four minutes* to work on this test.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. $88 + 54 + 79 =$ ☐

2. $611 - 335 =$ ☐

3. $37 \times 64 =$ ☐

4. $6.9 \times 5.2 =$ ☐

5. $98 \overline{)9016}$ ☐

6. $.39 \overline{)3.003}$ ☐

7. $5\frac{3}{4} + 8\frac{7}{8} =$ ☐

8. $5\frac{1}{3} - 2\frac{3}{4} =$ ☐

9. $3\frac{3}{4} \times 1\frac{1}{3} \div 1\frac{1}{4} =$ ☐

10. $7\frac{1}{2} \div 3 \times 1\frac{1}{5} =$ ☐

STOP. GO BACK AND CHECK YOUR WORK.

IV. NUMBER SERIES

Each problem consists of a series of six numbers formed according to some rule. You are to find the rule, and then write the next *two* numbers of the series on the lines at the right. EXAMPLES:

1.	12	12	9	9	6	6	3	3

In this example the rule was to write each number twice, and to subtract 3 from the number of each pair to get the number of the next pair.

2.	—2	4	—6	8	—10	12	-14	16

Here the rule was to add 2 to each number to get the next one, and to give a minus sign to every other number.

You will have *four minutes* to work on this test.

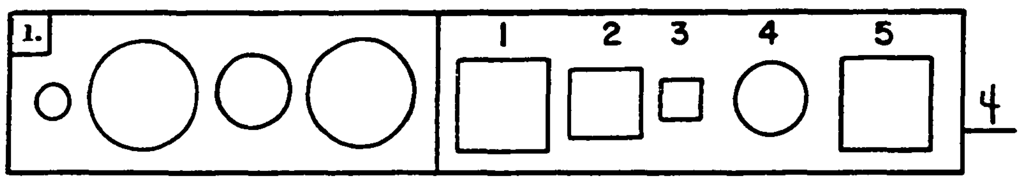
DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1.	3	3	2	2	1	1	____	____
2.	5	6	8	9	11	12	____	____
3.	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	____	____
4.	91	82	73	64	55	46	____	____
5.	10	9	7	4	0	—5	____	____
6.	63	48	35	24	15	8	____	____
7.	12	8	6	5	$4\frac{1}{2}$	$4\frac{1}{4}$	____	____
8.	625	$\frac{1}{625}$	125	$\frac{1}{125}$	25	$\frac{1}{25}$	____	____
9.	5	—7	10	—14	19	—25	____	____
10.	64	—49	—36	25	16	—9	____	____

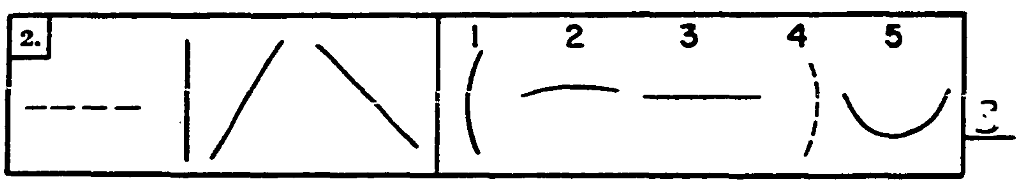
STOP. GO BACK AND CHECK YOUR WORK.

V. FIGURE CLASSIFICATION

In each problem there are four figures at the left, all alike in some way, and five numbered figures at the right. Find the *one* at the right which goes with those at the left, and write its *number* on the line at the far right of the box. EXAMPLES:



The figures at the left are round, and all but one of those at the right are square. Number 4 at the right is round, however, so it goes with those at the left, and a 4 has been written on the line at the far right.

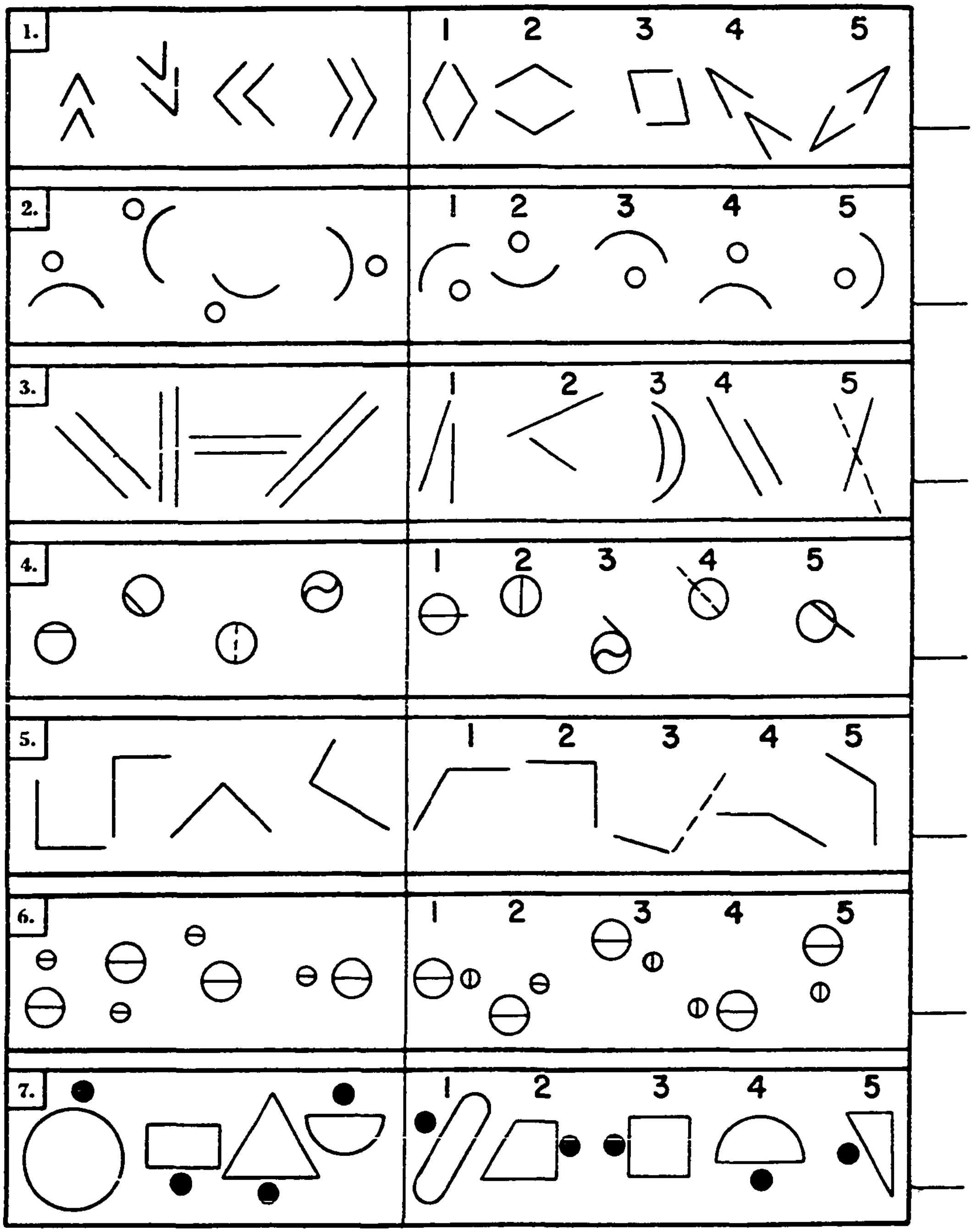


The answer is 3, because all the lines at the left are straight, and all but number 3 at the right are curved.

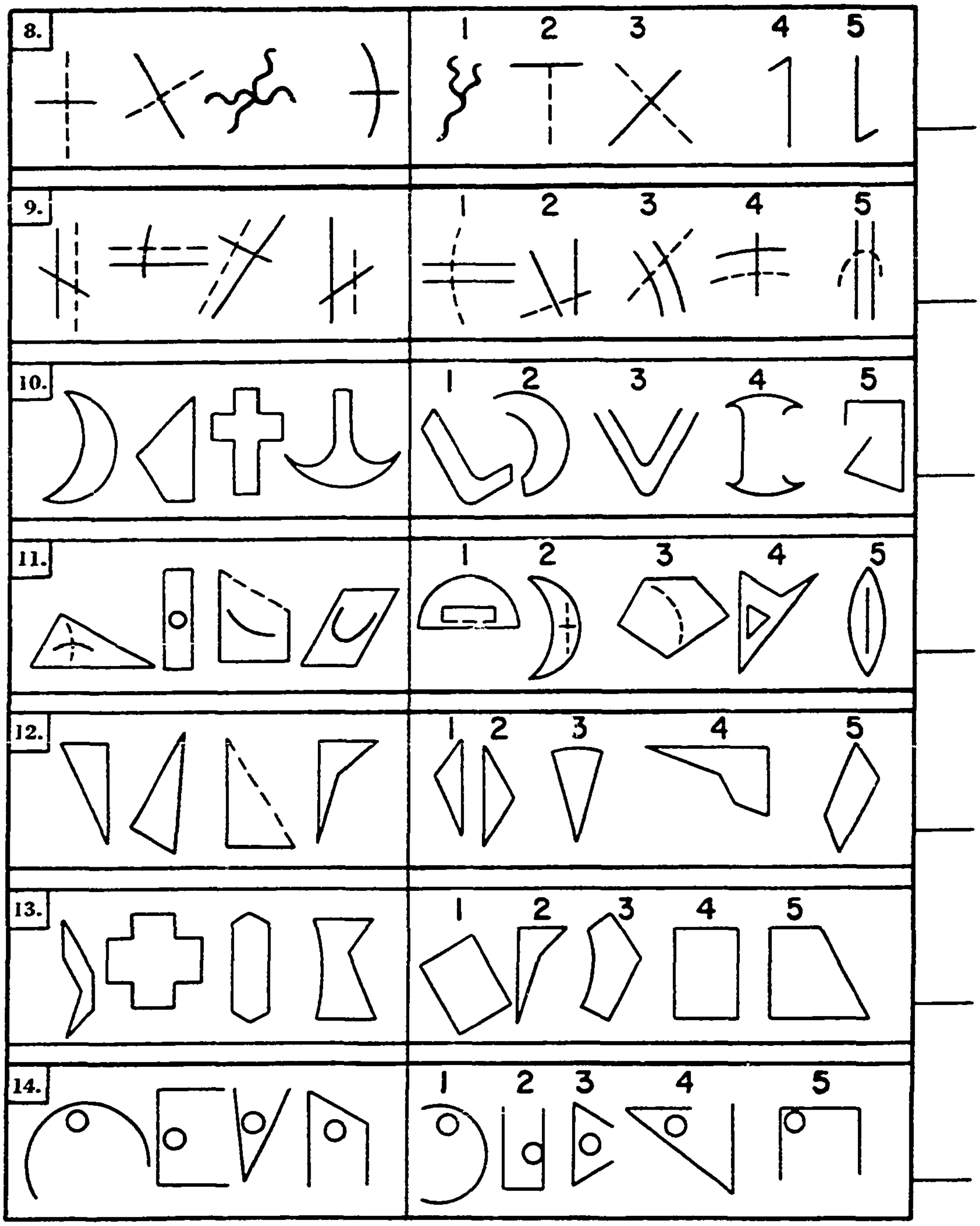
Mark an answer for every problem. If you don't know the answer to a problem, make the best choice you can.

You will have *four minutes* to work on this test.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.



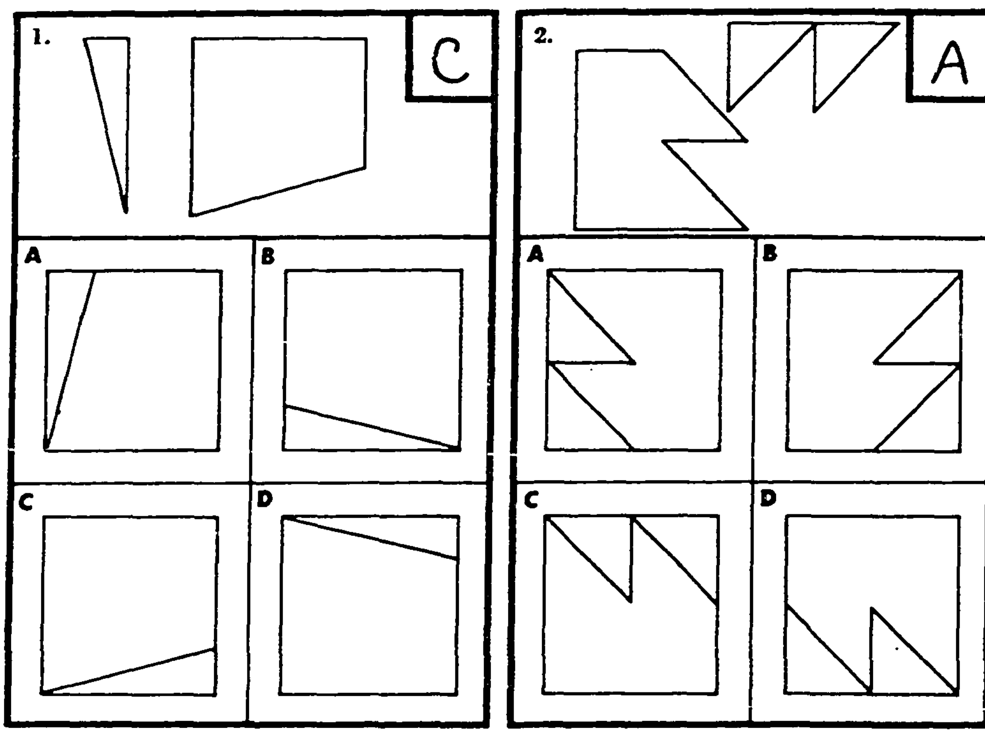
GO ON TO THE NEXT PAGE.



STOP. GO BACK AND CHECK YOUR WORK.

X. PAPER FORM BOARD

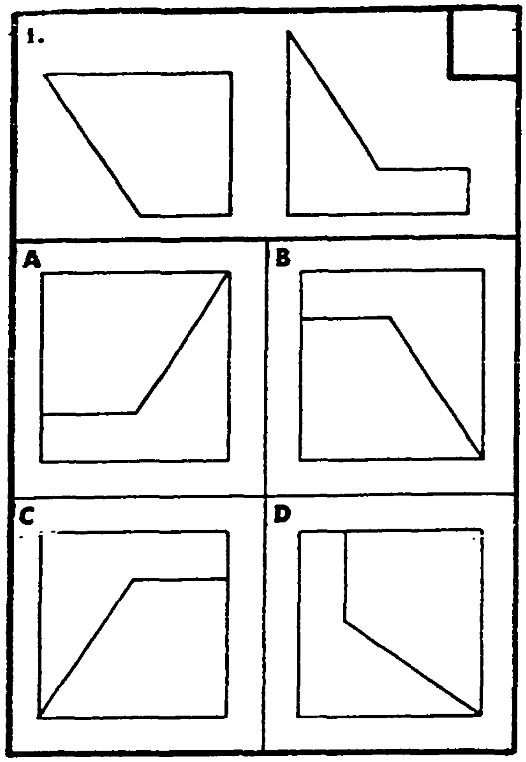
In each problem, think of the figures in the upper section as the pieces of a jigsaw puzzle. They can be slid around, but they *cannot* be turned over. If you slide them around, they will fit together to make one of the four figures, A or B or C or D, below. PRINT the CAPITAL LETTER of that figure in the upper right corner. EXAMPLES (with correct answers):

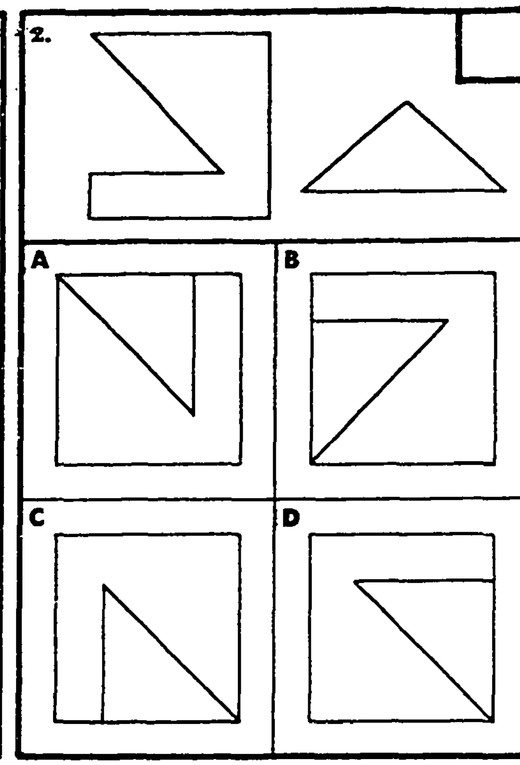


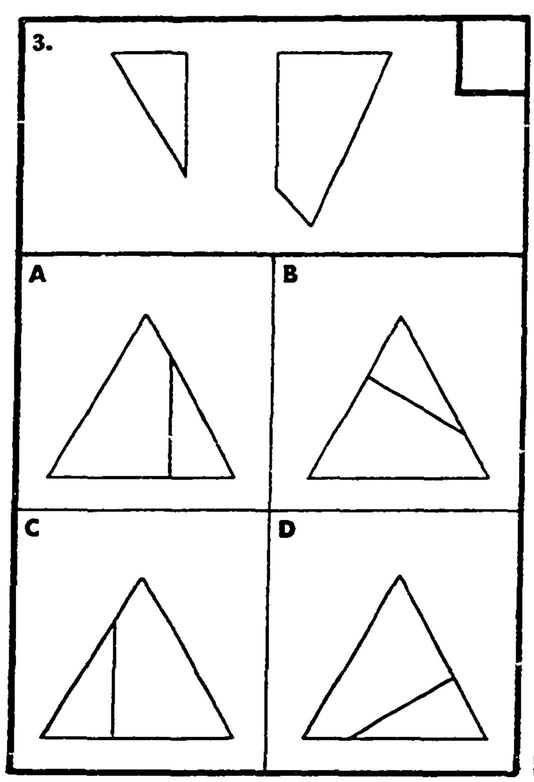
Mark an answer for every problem. If you don't know the answer to a problem, make the best choice you can.

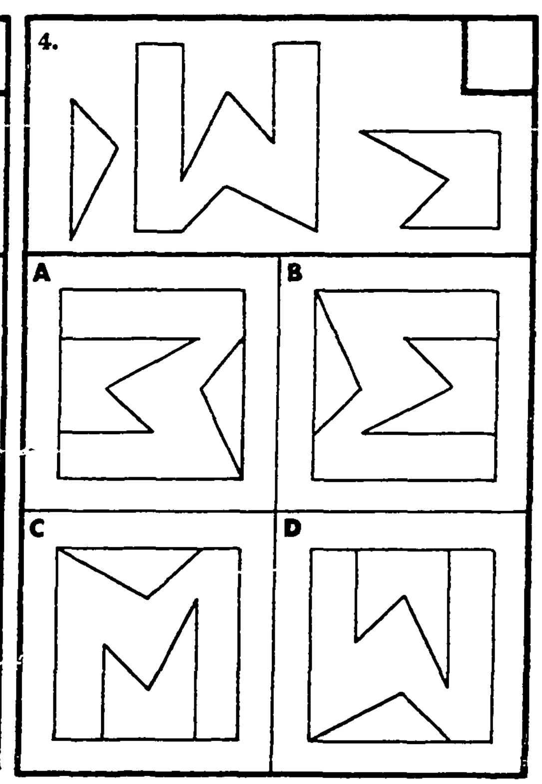
You will have *three minutes* to work on this test.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

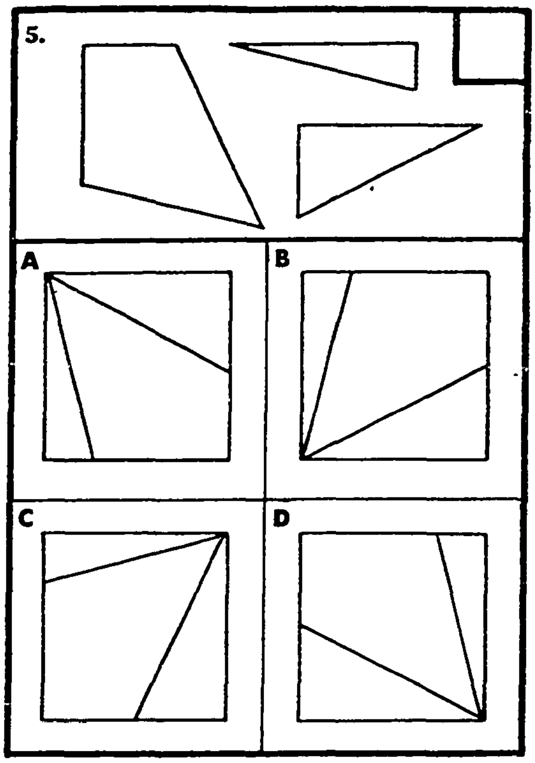


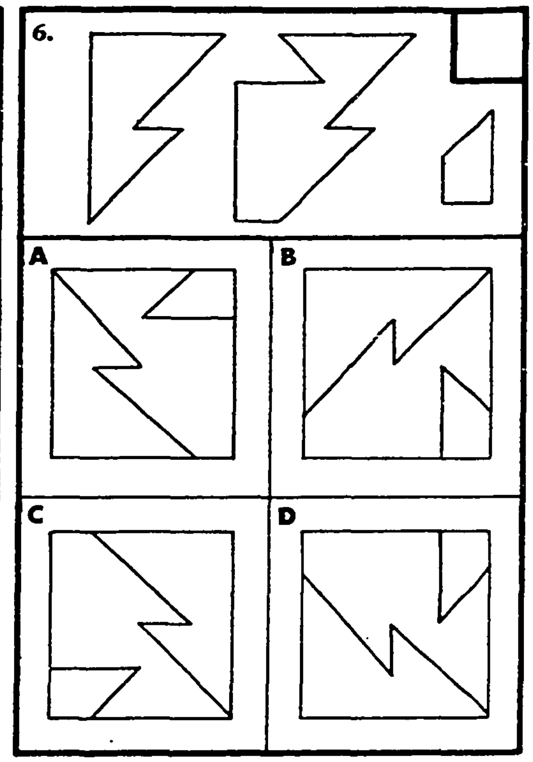


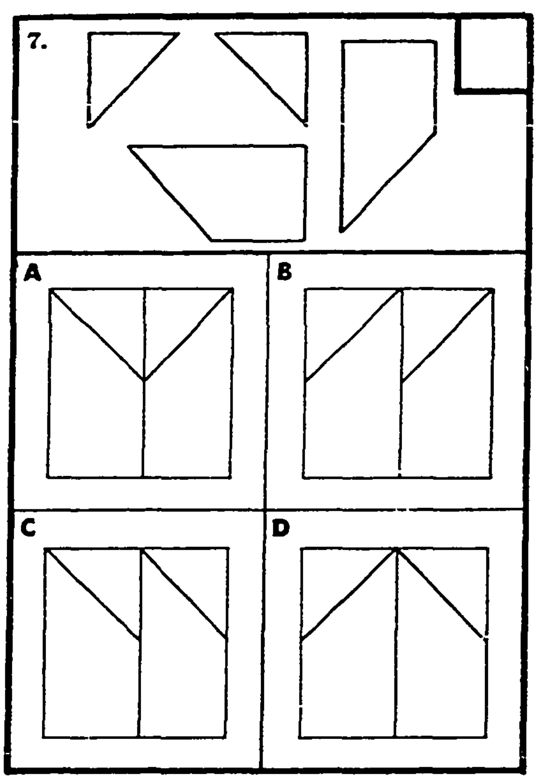


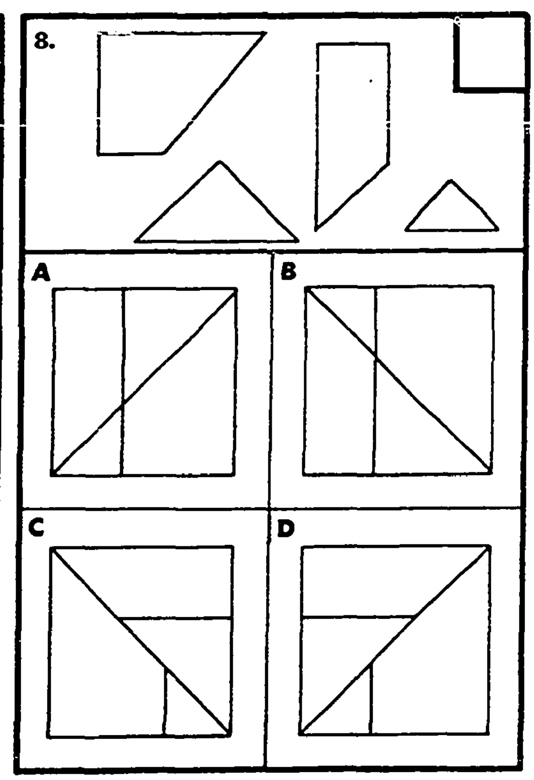


GO ON TO THE NEXT PAGE.









STOP. GO BACK AND CHECK YOUR WORK.

APPENDIX A.2: TEACHER RATING FORM

Dear Math Teachers,

For my research study on math aptitude, I need your help in rating the following students on their math aptitude. Each of the students listed below have taken both algebra 1-2 and geometry. Please find the names of the students you have taught for at least one semester and mark the appropriate space after their names. Also, most students' work reflects their aptitude but there are a few cases where a student has a higher aptitude than his grades and work reveal. For the purpose of identifying these students, please circle y=yes if you feel that the student's performance and aptitude were similar. Circle n=no if you feel that the student's performance was very different from his aptitude.

Thank you for your time and cooperation.

Student names	Math Aptitude HIGH — LOW	
1.	___:___:___:___:___:___:___	Y N
2.	___:___:___:___:___:___:___	Y N
3.	___:___:___:___:___:___:___	Y N
4.	___:___:___:___:___:___:___	Y N
5.	___:___:___:___:___:___:___	Y N
6.	___:___:___:___:___:___:___	Y N
7.	___:___:___:___:___:___:___	Y N
8.	___:___:___:___:___:___:___	Y N
9.	___:___:___:___:___:___:___	Y N
10.	___:___:___:___:___:___:___	Y N

APPENDIX A.3: EDWARDS PERSONALITY INVENTORY

DIRECTIONS: This inventory contains a number of statements that other people may or may not use in describing you. None of the statements are about your religious or political beliefs or your health. Every statement in the inventory is a statement that some person has used at one time or another in describing another person. Presumably, anyone who has observed you over a long period of time would be in a position to judge which of the statements in this inventory accurately describes you. Your task, in other words, is to predict how people who know you well would mark each statement if they were asked to describe you. Consider the following statement.

He is good at explaining things to others.

If you believe that people who know you well would say that this statement accurately describes you, then you should mark it true (A) on your answer sheet. If you believe they would say this this statement does not accurately describe you, then you should mark it false (B).

To mark your answers on the answer sheets, darken in A for true, B for false, and leave C, D, and E blank. Use answer sheet A (marked in green in the upper right-hand corner) for statements 1 to 100. Use answer sheet B for statements 101 to 190. On this second answer sheet place answer 101 on the page as if it were number 1. Put answer to 102 in number 2,... and 190 in number 90. Notice that the answers are numbered <u>across</u> the page, not down. Therefore, 5 is directly below 1.

Do <u>not</u> make any marks in the inventory booklet. Mark your answers only on the answer sheets.

While there is no time limit for taking this inventory, it is best to work as rapidly as possible. Do not spend a lot of time debating on the answers, and do not flip back and forth from page to page.

BEGIN by turning to the next page.

1. He always has his work carefully organized and planned.
2. He takes a great deal of pride in doing well whatever he does.
3. He has a tendency to use words that others don't know the meaning of.
4. He plays any game strictly according to the rules.
5. He will keep at a difficult task even when he has little hope of being successful.
6. He sets high standard of achievement for himself in his work.
7. He has imagination and analytical ability.
8. He spends so much time doing one thing very carefully that he has little time left for other things he should do.
9. He likes to try food he has never tasted before.
10. He is quite content to spend an evening alone watching television.
11. He regards doing the best he can as very important.
12. He is usually one of the last persons to finish an examination or test.
13. He is good at any work that requires careful attention to details.
14. He usually has his day's activities planned in advance.
15. He has a tendency to overlook important details in his work.
16. He can carry on an intelligent conversation about art and painting.
17. He is inclined to follow his own ideas rather than to do what is expected of him.
18. He puts in long hours of work without supervision.
19. He is not satisfied unless he has done the best job he knows how to do.
20. He has a talent for mathematics.
21. He is uncomfortable if he has to do a task imperfectly because of pressure of time.
22. He likes to work on a project where he has full responsibility for getting it done.

Go to next page.

23. He makes at least two or three drafts of any serious paper he writes.

24. He always checks any arithmetic he does several times to see if he has made a mistake.

25. He thinks a new way of doing something is almost always going to be better than the old.

26. He demands more of himself than others demand of him.

27. He has little interest in the small details of a problem.

28. He sets extremely high standards with respect to what he expects to accomplish in his work.

29. He becomes depressed if he is separated from his friend for any length of time.

30. He would hesitate to do anything that others might consider wrong.

31. He gets upset if he has four or five things requiring his attention at the same time.

32. He keeps his things arranged neatly and orderly on his desk or work space.

33. He is seldom satisfied with the first draft of a paper he has written.

34. He feels uncomfortable if he is asked to do something he has never done before.

35. He is the sort of person who would enjoy being a famous university professor.

36. He keeps at a job until it is completed.

37. He is a perfectionist in all of his work.

38. He has little interest in science.

39. He makes strong demands of himself.

40. He often wishes that something exciting would happen to him.

41. He likes to be alone with his thoughts whenever possible.

42. He will do something over and over again in an attempt to get it done right.

Go to next page.

43. He demands perfection in anything he undertakes.

44. He takes a great deal of pride in his work.

45. He has great admiration for anyone who has everything carefully organized.

46. He has a very high standard of work.

47. He enjoys seeing a serious stage play.

48. He does his best to prevent anyone from dominating him.

49. He seldom gets bored with what he is working on.

50. He is very fussy about minor things.

51. He has a good memory for the books he has read.

52. He seldom turns in a written assignment without first checking it for possible errors.

53. He is willing to accept the responsibility of making the plans for something that has to be done.

54. He is dedicated to his work.

55. He would rather not do a job than do it imperfectly.

56. He enjoys searching for new ways to do things.

57. He is the sort of person who makes you feel that if he does something it will be done right.

58. He is able to see the broader implications of a problem.

59. He is the sort of person who would enjoy doing research.

60. He has a number of hobbies he can work on alone that keep him busy during his spare time.

61. He is constantly seeking for the meanings and implications of things and ideas.

62. He is the sort of person others like to have with them in time of trouble.

63. He makes a plan before starting to do something difficult.

Go to next page.

64. He enjoys putting in long hours on his work.

65. He enjoys listening to a good opera.

66. He obeys without question rules and regulations set by those in a position of authority.

67. He forgets about time when he is working on something that interests him.

68. He seldom criticizes anyone in a position of authority.

69. He has a strong need to find out about things.

70. He has a great deal of interest in religion.

71. He sometimes does things that are dangerous just for the thrill of it.

72. He dislikes going to a movie by himself.

73. He enjoys being assigned to plan something.

74. He likes to do things that require the participation of others.

75. He frequently does something on impulse.

76. He sees to it that any written work he does is precise and well organized.

77. He is able to understand abstract ideas.

78. He owns a considerable number of books of a serious nature.

79. He does not like to be told what to do.

80. He will keep trying to solve a puzzle long after everyone else has given up.

81. He has difficulty putting aside a book he is interested in once he has started reading it.

82. He admires people who are original and unconventional.

83. He is uncomfortable in any situation in which he does not know clearly what is expected of him.

84. He has confidence in his ability to get things done.

85. He seldom reads a novel.

Go to next page.

86. He sticks to a plan of action once he has decided on it.

87. He is changeable in his likes and dislikes.

88. He thinks most things out for himself.

89. He makes the best possible use of his abilities.

90. He likes to take walks by himself.

91. He enjoys doing things on the spur of the moment.

92. He would rather be an administrator than a researcher.

93. He is not sure of his opinions.

94. He plans his work carefully.

95. He reads only things that interest him.

96. He likes going to symphonies and concerts.

97. He can always find some reason for doing what he wants to do.

98. He does not mind staying up late in order to get a job done.

99. He makes up his mind quickly about things.

100. He is interested in finding out how things work.

101. He gets little enjoyment from an intellectual problem or task.

102. He buys things he can't really afford.

103. He doesn't depend on the company of others to keep from being bored.

104. He has been known to stay up all night working on something he was interested in.

105. He has a strong need to be independent of others.

106. He often picks up a book and reads it from beginning to end without stopping.

107. He carefully outlines any speech or paper he has to give.

108. He believes that most problems can be solved if the approach to them is carefully planned.

Go to next page.

109. He has a large vocabulary that he doesn't hesitate to use in talking to others.

110. He does things that others regard as unconventional.

111. He has a tendency to leave tasks unfinished.

112. He has no desire to have many close personal friends.

113. He enjoys thinking and speculating about a problem.

114. He has been known to plan carefully a holiday or vacation and then decide to do something else at the last minute.

115. He dislikes being forced to make a decision on the spur of the moment.

116. He enjoys reading about developments in modern science.

117. He likes to put in long hours of work without distraction.

118. He likes to experiment and try new things.

119. He is a very conventional person.

120. He tends to identify himself with the characters in the movies he sees.

121. He made plans to attend college when he was a freshman in high school.

122. He is self-confident in whatever he does.

123. He is very much interested in science.

124. He has the ability to make sound judgments.

125. He likes to have things organized down to the last detail.

126. He writes comments in the margins of the books he owns.

127. He likes to read poetry.

128. He resents having to conform to the rules and regulations of a group he belongs to.

129. He is able to concentrate on a single task for long periods of time.

130. He doesn't seem to be able to enjoy himself when he is alone.

Go to next page.

131. He asks intelligent questions about things he doesn't understand.

132. He is the sort of person who changes his opinions and attitudes from day to day.

133. He will try almost anything once.

134. He likes to work on a project by himself.

135. He can concentrate on reading a book even while others carry on a conversation.

136. He spends considerable time trying to improve his knowledge of things.

137. He does his best to avoid being interrupted at his work.

138. He likes to follow a set plan in doing his work.

139. He conforms to custom.

140. He is the sort of person who would have difficulty liking any kind of modern art.

141. He has a streak of nonconformity in him.

142. He keeps working at a puzzle or a problem until he solves it.

143. He would never voluntarily go to an art gallery.

144. He enjoys reading a book from which he learns something.

145. He is critical of people who start in to do something without first planning what they are going to do.

146. He hates to work under pressure.

147. He is happiest when relaxing with a group of friends.

148. He enjoys dining in some restaurant where he has not been before.

149. He is usually one of the first to participate in any new fad or fashion.

150. He likes to read science fiction.

151. He is disturbed if he is not immediately successful in learning a new skill.

Go to next page.

152. He has accepted most of the beliefs and values of his parents.

153. He is considered by others as an expert on good books.

154. He is more intelligent than most people.

155. He has difficulty making an important decision without help from others.

156. He carefully plans what he proposes to say in a group meeting.

157. He has his daily activities planned so that he knows just what he will be doing at any given time.

158. He considers foreign motion pictures superior to the average Hollywood production.

159. He usually manages to do what he wants to do.

160. He gets so involved in his work that he neglects his sleep.

161. He is sure of himself in most situations.

162. He places a high value on intellectual achievement.

163. He refuses to accept the notion that there simply aren't any answers for some questions.

164. He has a reputation for doing unpredictable things.

165. He is completely happy spending an evening alone reading an interesting book.

166. He understands something better by studying it alone than be discussing it with others.

167. He welcomes any interruption in his daily routine.

168. He shows considerable initiative in accepting responsibility.

169. He plans and organizes the details of any complicated tasks he undertakes.

170. He doesn't like to talk about anything that is theoretical in nature.

171. He would enjoy being a famous musician.

172. He avoids doing things that other people might consider unconventional.

Go to next page.

173. He has difficulty setting aside a task he has undertaken even for a short time.

174. He attacks a task with energy and a strong desire to accomplish it.

175. He would never voluntarily read a technical book.

176. He has a reputation for being a nonconformist.

177. He does not resent having to assume responsibility for the actions of others.

178. He would enjoy being a famous composer.

179. He likes to have things organized.

180. He can usually be counted on to suggest something new to do when in a group.

181. He has periods during which he wants to be alone.

182. He spends most of his spare time doing things with others.

183. He likes to travel.

184. He enjoys thinging about a serious problem.

185. He is primarily interested in concepts and ideas.

186. He is seldom nervous when faced with a new situation.

187. He enjoys planning the details of his vacation or holiday.

188. He has a strong need to complete whatever he undertakes.

189. He has a wide range of interests.

190. He resents rules and regulations.

Check your answer sheet for incomplete items and complete them. Do NOT go on to the next test.

APPENDIX A.4: PROBLEM SOLVING TEST

DIRECTIONS: This problem solving test is divided into two parts. The first part is a questionnaire about your musical experience and your preferences in problem solving situations. The second part consists of five problems to be worked out. Some of the problems will require you to use either algebraic or geometric methods. For these, choose the method that is easier for you. You may work the problems in any order. It is important that you attempt all of the problems, so do not spend too much time on any one problem. You may work on the next page.

After completing the problem, rate yourself on the scale at the bottom of the page. The statement will appear as:

I think my answer to this problem is
correct ___:___:___:___:___ incorrect.

Place an X in the space that indicates the degree of confidence you have in your answer. An X mark in the middle space implies that you believe your answer has a 50-50 chance of being correct.

You will have 45 minutes in which to complete this test. DO NOT BEGIN UNTIL TOLD TO DO SO.

PART I

MUSICAL EXPERIENCE

DIRECTIONS: Complete the following statements by circling the number to the appropriate response(s) in each category below.

1. I have played a musical instrument for over one year. 1=true 2=false
   If you circled 2, go on to question 3.

2. I play (have played) the following types of musical instruments:

A. Type	B. Number of years I have played	C. Organization
1 brass	1 one to three	1 orchestra
2 percussion	2 four to six	2 band
3 strings	3 seven or more	3 small group
4 winds		4 private

3. I have had vocal instruction for over one year. I true 2 false
   If your circled 2, skip question 4 and go on to next section below.

4. I am taking (have taken) vocal instruction with

A. Organization	B. Number of years (total)
1 school chorus	1 one to three
2 church group	2 four to six
3 private lessons	3 seven or more

PROBLEM SOLVING PREFERENCES

DIRECTIONS: Underline the phrase(s) that describe(s) your performance or behavior.

1. When a problem can be solved by either algebraic or geometric methods, I prefer to use the (algebraic, geometric) approach.

2. When solving mathematical problems I (always, usually, seldom, never) find short cuts.

3. After having worked on a difficult problem and not getting anywhere, I usually (give up, leave it and return later, look at the answer, try to work it backwards, work on it some more, check the directions).

4. I (do, do not) like to work problems that have little practical application.

5. When working word problems I usually (draw a diagram, write the numerals down, make up an equation as I read the problem, do not know where to begin).

Go to next page.

6. I prefer puzzles having (a definite scene, an abstract design).

7. I think my mathematical aptitude is (very high, above average, average, below average, very low).

8. In my spare time I solve problems using mathematics (frequently, occasionally, seldom, never).

9. I work jigsaw puzzles in the following manner. (1=first step, 2=second step...) Mark only the steps you use.

___find the edges ___separate the colors ___separate the shapes

___check the picture ___turn the pieces face up ___measure the dimensions

PART II

1. MISSING VALUES

SITUATION: You are given a set of four equations with four unknowns.

TASK: Find the numerical values for A, B, and C. Do your work on this page.

GIVEN: $A = B - 2C$

$B = (\frac{1}{2})\ CD$

$C = A + 2B + 30\ D$

$D = -2$

SOLUTION: A = _______, B = _______, C = _______.

I think my answer to this problem is

correct___:___:___:___:___ incorrect.

2. SCHOOL LOCATION AND BUS ROUTES

SITUATION: A consolidated school is to be built in the rural district shown in the diagram. The letters A, B,..., G indicate points where students are to be picked up by two school buses. The capacity of each bus is 11 students and 1 bus driver. Mileage between points is shown below.

TASK: Find the best routes for the school buses; that is, name the points along the routes where the following conditions are met

1) the buses can pick up all students without overloading;
2) the routes do not overlap; and
3) the routes require the minimum amount of miles.

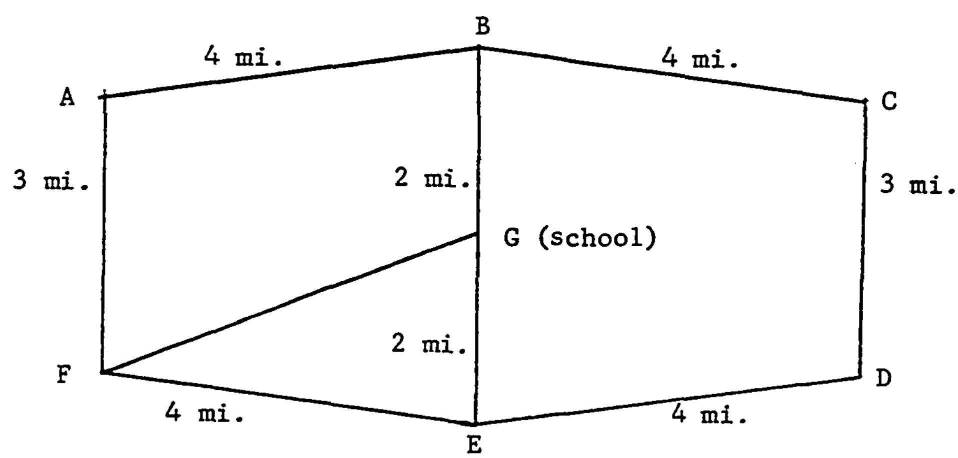
The buses may start at any point and need not begin from the school each morning.

GIVEN: The school is at point G.

Pickup points:	A	B	C	D	E	F
Number of students:	5	4	3	1	2	3

SOLUTION: One of the buses begins at point _____. Its morning route goes through points:___________.

The second bus starts at point_____. Its morning route goes through points:___________.



I think my answer to this problem is

correct ___:___:___:___:___ incorrect.

3. ANT WANTS THE FOOD

SITUATION: An ant is located at the upper left-hand corner of a card-board box (lid attached). The ant wants to get the food which is at the lower right-hand corner by the shortest possible route.

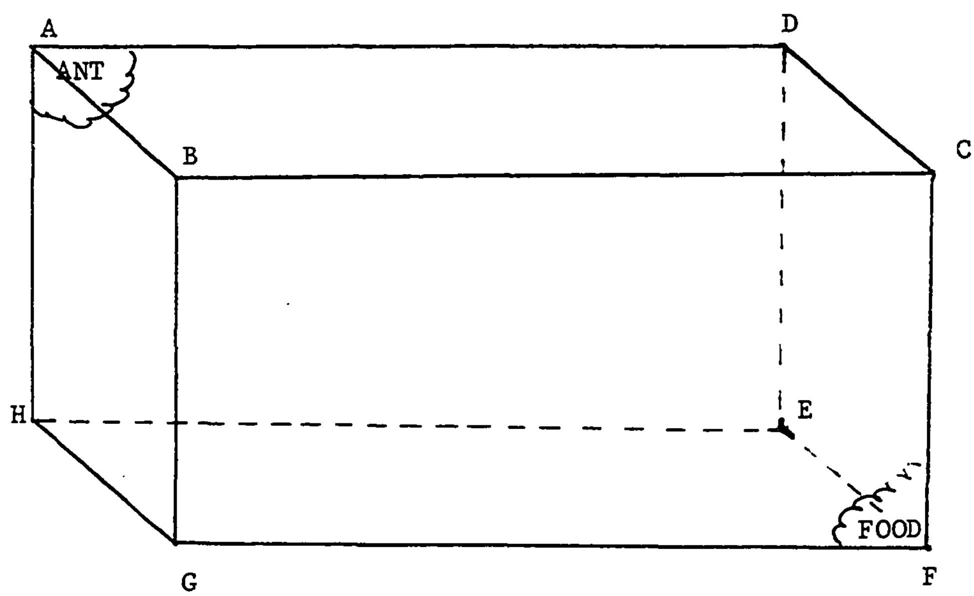
TASK: Solve this problem either 1) geometrically, by drawing in the shortest possible route the ant could take, OR 2) algebraically, by finding the equation which represents the shortest possible route the ant could take.

RULE: The ant must arrive at the corner with the food by walking, not falling nor travelling through space.

GIVEN: All faces are rectangles. $\overline{AB}$ = 1 ft., $\overline{AH}$ = 2 ft., $\overline{AD}$ = 4 ft.

SOLUTION: Algebraic equation________________________________.

Geometric solution (drawn in below).



I think my answer to this problem is

correct ___:___:___:___:___ incorrect.

4. CLASS COUNT

SITUATION: The school counselors wanted to know how many students wanted to take chemistry, German, and algebra II during the next school year. There were 113 students who sent back their questionnaires.

TASK: Find the number of students who wanted to take all three courses.

GIVEN:

Number of students	Course(s)
90	algebra II
60	chemistry
20	German
40	algebra II and chemistry
16	algebra II and German
1	chemistry and German ONLY
19	chemistry ONLY
6	algebra II and German ONLY

SOLUTION: The total number of students who wanted to take all three courses was _______.

I think my answer to this problem is

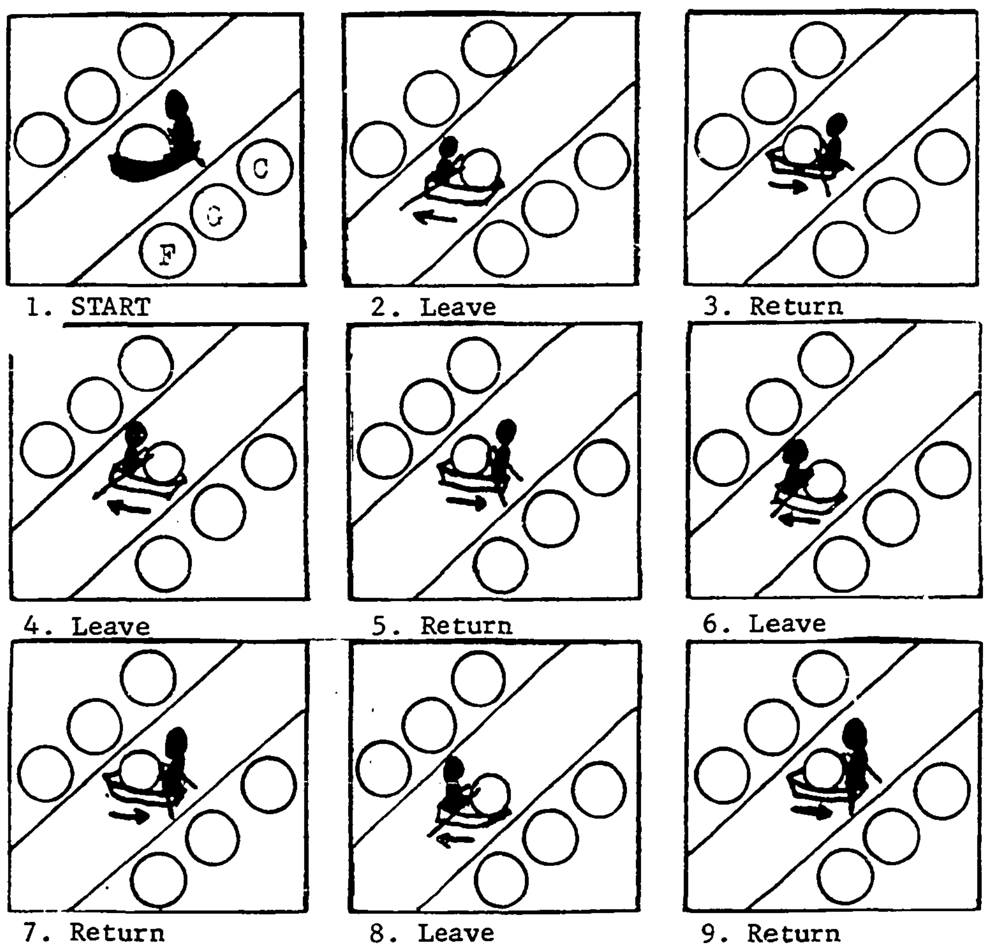
correct ___:___:___:___:___ incorrect.

5. THE FOX, THE GOOSE, AND THE BAG OF CORN

SITUATION: A farmer must row a fox, a goose and a bag of corn across the river. The boat is so small that there is room for only one "passenger" and the farmer at any one time.

TASK: Using the following abbreviations (F = fox, G = goose, C = corn), place the letter of the passengers in the appropriate circles below.

NOTE: Each frame represents one step. Leave extra circles and frames blank. Frame 1 shows the starting positions. You are to begin with frame 2. You are to use the other frames to show how the farmer would get his possessions to the opposite side of the river without loss or damage to them.



I think my answer to this problem is

correct ___:___:___:___:___ incorrect.

APPENDIX A.5: SEMANTIC DIFFERENTIAL INTEREST INVENTORY

DIRECTIONS: The purpose of this instrument is to measure the meanings of certain words to various students by having you judge the words against a series of descriptive scales. In taking this test, make your judgments on the basis of what these words mean to you. On each page you will find a different concept to be judged and beneath it a set of scales. You are to rate each concept on the scales in order.

Examples of how to use these scales:

1. A mark (X) at the end space means that you feel the concept (in the case below, Biology) is VERY CLOSELY RELATED.

BIOLOGY

interesting_X_:___:___:___:___:___:___boring

OR

interesting___:___:___:___:___:___:___boring

2. A mark (X) near the end space means that you feel the concept at the top of the page is QUITE CLOSELY RELATED.

BIOLOGY

interesting___:_X_:___:___:___:___:___boring

OR

interesting___:___:___:___:___:_X_:___boring

3. A mark (X) toward the middle space means that you feel the concept at the top of the page is ONLY SLIGHTLY RELATED.

BIOLOGY

interesting___:___:_X_:___:___:___:___boring

OR

interesting___:___:___:___:_X_:___:___boring

4. A mark (X) in the middle space means that you feel the concept is either NEUTRAL on the scale, both sides of the scale are EQUALLY ASSOCIATED with the concept, or the scale is COMPLETELY IRRELEVANT, UNRELATED to the concept.

BIOLOGY

interesting___:___:___:_X_:___:___:___boring

NOTE: The direction toward which you mark depends upon which of the two ends of the scale seem most characteristic of the concept you are judging.

IMPORTANT: (1) Place your X marks in the middle of the spaces, not on the boundaries.

DO THIS___:_X_:___ NOT THIS___:__X__

(2) Be sure to mark every scale for every concept.... do not omit any.

(3) Never put more than one mark on a single line.

Do not look back and forth through the items and do not try to remember how you marked similar items appearing earlier in the test. Make each item a separate and independent judgment. Work quickly and do not puzzle over individual items. It is your first impressions, the immediate feelings about the items, that are asked for. However, please do be careful to try to give your true impressions.

Do not begin until you are instructed to do so.

MATHEMATICS

pleasant____:____:____:____:____:____:____unpleasant

bad____:____:____:____:____:____:____good

active____:____:____:____:____:____:____passive

valuable____:____:____:____:____:____:____worthless

weak____:____:____:____:____:____:____strong

love____:____:____:____:____:____:____hate

slow____:____:____:____:____:____:____fast

uncomfortable____:____:____:____:____:____:____comfortable

nice____:____:____:____:____:____:____awful

enjoyable____:____:____:____:____:____:____unenjoyable

varied____:____:____:____:____:____:____repetitive

hard____:____:____:____:____:____:____soft

unafraid____:____:____:____:____:____:____afraid

light____:____:____:____:____:____:____heavy

insecure____:____:____:____:____:____:____secure

Go to next page. M

READING

soft____:____:____:____:____:____:____hard

valuable____:____:____:____:____:____:____worthless

fast____:____:____:____:____:____:____slow

unenjoyable____:____:____:____:____:____:____enjoyable

insecure____:____:____:____:____:____:____secure

pleasant____:____:____:____:____:____:____unpleasant

afraid____:____:____:____:____:____:____unafraid

strong____:____:____:____:____:____:____weak

uncomfortable____:____:____:____:____:____:____comfortable

heavy____:____:____:____:____:____:____light

bad____:____:____:____:____:____:____good

active____:____:____:____:____:____:____passive

hate____:____:____:____:____:____:____love

nice____:____:____:____:____:____:____awful

repetitive____:____:____:____:____:____:____varied

Go to next page. R

ALGEBRA

good____:____:____:____:____:____:____bad

insecure____:____:____:____:____:____:____secure

worthless____:____:____:____:____:____:____valuable

love____:____:____:____:____:____:____hate

enjoyable____:____:____:____:____:____:____unenjoyable

soft____:____:____:____:____:____:____hard

passive____:____:____:____:____:____:____active

unpleasant____:____:____:____:____:____:____pleasant

light____:____:____:____:____:____:____heavy

slow____:____:____:____:____:____:____fast

awful____:____:____:____:____:____:____nice

strong____:____:____:____:____:____:____weak

afraid____:____:____:____:____:____:____unafraid

comfortable____:____:____:____:____:____:____uncomfortable

repetitive____:____:____:____:____:____:____varied

Go to next page.

A

GEOMETRY

nice____:____:____:____:____:____:____awful

active____:____:____:____:____:____:____passive

unafraid____:____:____:____:____:____:____afraid

repetitive____:____:____:____:____:____:____varied

unpleasant____:____:____:____:____:____:____pleasant

comfortable____:____:____:____:____:____:____uncomfortable

strong____:____:____:____:____:____:____weak

bad____:____:____:____:____:____:____good

soft____:____:____:____:____:____:____hard

secure____:____:____:____:____:____:____insecure

unenjoyable____:____:____:____:____:____:____enjoyable

hate____:____:____:____:____:____:____love

heavy____:____:____:____:____:____:____light

fast____:____:____:____:____:____:____slow

valuable____:____:____:____:____:____:____worthless

Go to next page.

G

APPENDIX A.6: CONTENT VALIDITY STUDY OF PROBLEM SOLVING TEST

The following seven problems have been selected for a problem solving test to be given to high school juniors. These students will have completed both algebra I and geometry. Your cooperation and suggestions are solicited for the content validation. The problems and directions are given to you as they will appear for the students. The solutions and scoring are included after the test. Please study one problem at a time, rate it on the sheet below, and then continue to the next item. Item difficulty will be on a 5-point scale (1 = very easy, 5 = very difficult). All other concepts are to be rated on the degree of the concept's presence within the item. These will be on a 3-point scale (1 = incidental to the problem, 2 = moderate role in the problem, 3 = essential to the problem). A blank indicates the concept does not appear in the problem.

Concepts	Problem Number						
	1	2	3	4	5	6	7
Item difficulty (1 to 5)							
Arithmetic skills							
Algebraic skills							
Geometric skills							
Logical reasoning							
verbal ability							
nonverbal reasoning							
spatial perception							
flexibility							
application							
accuracy							
persistence							
trial & error							

APPENDIX B: TABLES

Table B.1. Description of sample by school and sex

	Male	Female	Total
School 1	67	48	115 (38%)
School 2	90	95	185 (62%)
Total	157 (52%)	143 (48%)	300 (100%)

Table B.2. Description of sample by parents' education level

Parent	Elementary	Junior high	High school	Some college	B.A. degree	graduate	n[a]	$\bar{X}$	S
Mother	3	21	103	28	18	2	175	12.46	1.83
Father	9	19	92	18	26	14	178	12.80	2.49

[a]Number of cases is less than 300 due to missing data.

Table B.3. Description of sample by parents' socioeconomic status (SES)

Code[a]	SES	Mother	Father
90	Professional, technical	3	31
81	Manager, official, proprietor	31	35
71	Clerical, sales	28	97
58	Craftsmen, foremen	8	49
45	Operatives	12	13
34	Service workers	6	30
20	Laborers (not farm)	4	27
16	Farmers, farm workers	--	12
06	Farm laborers, foremen	--	1
00	Housewives	219	--
	n	300	295
	$\bar{X}$	19.36	63.32
	S	33.18	23.70

[a]U.S. Dept. of Census, HEW, 1950.

Table B.4. Comparison of sample to nonparticipants[a]

Measure	Sample = 300			Nonparticipants = 71			t	df
	$\bar{X}$	S	n	$\bar{X}$	S	n		
IQ	113.31	10.59	292	110.38	9.48	68	0.31	360
Sex								
male	--	--	157	--	--	33	--	--
female	--	--	143	--	--	38	--	--
Class rank	.24	.21	265	.28	.24	58	-0.21	323
MED	12.46	1.83	175	11.94	2.17	32	0.31	207
FED	12.80	2.49	178	14.12	4.04	33	-1.94	211
MOCC	19.36	33.18	300	19.78	31.08	71	-0.01	371
FOCC	63.32	23.70	295	58.10	24.99	71	0.25	366
MGPA	2.64	0.85	300	2.44	0.79	71	0.26	371
SCON	16.57	4.74	286	15.57	4.94	67	0.23	353
NSBK	18.16	4.98	286	17.39	4.40	66	0.17	352
EXPR	17.01	3.75	286	15.83	4.45	65	0.38	351
QUANT	17.53	5.53	287	15.84	4.94	66	0.34	353
SSREAD	17.36	5.92	286	16.13	5.37	64	0.24	350
NSREAD	18.10	6.28	287	16.57	5.12	65	0.27	352
LIT	16.79	5.72	286	15.80	5.28	65	0.19	351
VOCAB	18.02	4.19	286	17.30	4.42	66	0.23	352
USES	18.33	5.18	286	16.88	5.45	65	0.31	351

[a] $t_{200} = 1.97$, $(p < .05)$; $t_{300} = 1.97$, $(p < .05)$.

Table B.5. Distribution of sample by math grade point average (MGPA) and teacher rating (TR)

MGPA[a]	Teacher rating (low) 1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	(high) 7.0	Total	No TR
4.00	-	-	-	-	-	-	-	-	3	1	7	6	4	21	-
3.75	-	-	-	-	-	-	-	-	2	4	5	3	1	15	1
3.50	-	-	-	-	1	-	1	1	4	6	6	2	6	27	3
3.25	-	-	-	-	-	-	2	2	7	6	7	1	-	25	2
3.00	-	-	-	1	-	1	4	7	7	1	7	3	4	35	3
2.75	-	-	-	-	-	3	2	3	7	8	2	-	4	29	3
2.50	-	-	1	-	4	3	7	3	4	1	3	-	2	28	3
2.25	-	-	-	-	-	3	7	1	5	1	1	1	1	20	3
2.00	1	-	1	-	3	1	7	-	1	-	1	-	-	15	-
1.75	-	-	-	2	5	3	11	-	2	-	-	-	-	23	2
1.50	-	-	1	1	7	1	4	-	-	-	-	-	-	14	2
1.25	-	-	2	-	5	-	3	-	1	-	-	-	-	11	1
1.00	-	-	2	1	2	1	-	-	-	-	-	-	-	6	2
0.75	-	-	1	1	1	1	-	-	-	-	-	-	-	4	1
0.50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.25	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-
0.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	r = .722										Total			274	26

[a]MGPA is average of four semester grades; two algebra grades and two geometry grades.

Table B.6. Distribution of sample by algebra grade point average (AGPA) and geometry grade point average (GGPA)

AGPA[a]	GGPA[b]								
	0.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00
4.00			1	1	1	4	8	5	21
3.50				1	3	7	8	6	11
3.00				2	9	8	24	12	16
2.50			1	2	9	9	14	2	3
2.00		3	6	10	11	9	9	6	4
1.50		3	5	3	13	2	3	2	
1.00		1	4	4	7	1			
0.50			1	3					
0.00			1		1				

[a]Algebra GPA is average of two semester grades in algebra.

[b]Geometry GPA is average of two semester grades in geometry.

Table B.7. Interrater correlations of content ratings on PST[a]

Rater																	$\bar{X}$	s
1																	1.48	1.14
2	34																1.33	1.16
3	41	59															0.60	0.96
4	26	37	51														1.60	1.46
5	38	52	77	48													0.70	1.12
6	58	43	40	22	17												1.58	1.09
7	46	43	71	47	64	43											1.55	0.67
8	49	30	44	30	30	50	49										1.08	1.00
9	44	34	43	28	37	41	62	39									2.00	0.61
10	32	20	48	57	41	35	54	48	35								1.02	1.27
11	41	27	41	38	39	28	52	42	44	36							1.95	0.81
12	39	40	55	42	54	33	61	47	39	46	31						1.07	1.15
13	52	24	41	31	45	42	56	44	51	48	48	51					0.88	1.25
14	53	50	44	36	35	43	45	35	56	33	27	46	33				1.97	0.84
15	43	56	43	42	42	46	47	44	62	27	43	39	28	51			2.23	0.77
16	35	46	43	47	50	10	30	21	17	23	27	36	14	31	52		1.50	1.11
17	43	56	48	45	48	53	50	50	34	45	28	41	33	42	53	16	1.20	1.22

$\bar{r}$ = .42 Coef. α = .92 Scale: 0 to 3

[a]Decimals omitted in correlation matrix.

Table B.8. Description of content of PST by item[a] (n = 17 raters)

Concept	1-Equa $\bar{X}$	1-Equa S	2-Bus $\bar{X}$	2-Bus S	3-Ant $\bar{X}$	3-Ant S	4-Venn $\bar{X}$	4-Venn S	5-Fox $\bar{X}$	5-Fox S
Arithmetic skills	1.88	0.93	1.88	0.86	1.41	0.94	1.82	1.02	0.35	0.49
Algebraic skills	3.00	0.00	0.65	0.79	2.47	0.80	1.06	1.03	0.24	0.44
Geometric skills	0.24	0.44	1.18	0.95	2.71	0.47	1.06	1.20	0.35	0.70
Logical reasoning	1.24	1.09	2.35	0.49	2.35	0.86	2.71	0.47	2.77	0.44
Verbal ability	0.65	0.79	1.24	0.97	1.18	0.95	1.71	0.85	1.59	0.21
Nonverbal ability	1.29	1.11	1.53	1.13	1.94	1.20	1.81	1.17	1.53	1.38
Spatial perception	0.29	0.47	1.24	1.03	2.65	0.49	0.82	1.07	0.77	0.97
Flexibility	0.59	0.71	1.59	0.94	1.59	1.23	0.71	0.77	1.53	1.07
Application	0.94	1.14	1.12	1.11	1.24	1.20	1.12	1.22	1.18	1.13
Accuracy	2.29	1.05	1.12	1.05	1.65	1.06	1.88	1.17	0.88	1.11
Persistence	1.06	1.20	1.53	1.01	0.88	1.11	0.94	0.90	1.94	1.09
Trial and error	0.65	0.79	1.94	1.03	0.71	0.85	0.71	0.69	2.06	1.14

[a]Rating scale: 3 - essential role
2 - moderate role
1 - incidental
0 - does not appear.

Table B.9. Interrater correlations on item difficulty of Problem Solving Test[a]

Rater																	$\bar{X}$	S
1																	3.8	1.30
2	-27																4.0	0.71
3	18	-42															2.8	0.84
4	26	-81	87														3.8	1.30
5	-51	0	53	34													2.8	0.45
6	51	0	13	09	25												3.4	0.89
7	-50	0	64	41	53	-53											3.8	0.84
8	47	29	-73	-63	-91	0	73										3.0	1.22
9	-51	0	53	34	100	25	53	-91									3.8	0.45
10	38	-35	90	77	56	56	30	-61	56								3.0	1.00
11	-05	-85	29	64	-13	-53	29	-24	-13	0							3.8	0.84
12	96	0	0	0	-56	56	-60	61	-56	25	-30						4.0	1.00
13	76	0	50	32	09	77	-18	0	09	77	-41	77					3.2	1.30
14	-13	-40	13	30	-38	-88	47	0	-38	-28	80	-28	-51				2.4	0.89
15	87	0	29	18	-13	80	-43	24	-13	60	-43	90	96	-53			3.8	0.84
16	-34	-79	13	51	25	-25	13	-46	25	0	80	-56	-51	38	-53		3.4	0.89
17	14	-65	22	49	41	61	-33	-37	41	46	22	0	21	-41	22	61	3.4	0.55

$\bar{r}$ = .08 Coef. alpha = .59 Scale: 1 to 5

[a]Decimals omitted in correlation matrix.

Table B.10. Raters estimation of item difficulty of PST

	1-Equa	2-Bus	3-Ant	4-Venn	5-Fox
$\bar{X}$	3.41	3.35	3.77	3.71	2.88
S	0.94	1.00	1.09	0.77	0.86
p (item difficulty)	0.32	0.33	0.25	0.26	0.42
$\bar{p}$ = .315	n = 17 judges				

Table B.11. Correlations and reliability coefficient of the trial form of PST

Problem								$\bar{X}$	S	p
1 Sort	Ø							3.00	0.00	1.00
2 Bus	Ø	—						2.46	0.80	0.82
3 Ant	Ø	.24	—					1.68	0.78	0.56
4 Rocket	Ø	.22	.13	—				0.27	0.88	0.09
5 Fox	Ø	.04	-.01	.16	—			0.82	1.37	0.27
6 Equa	Ø	.02	.13	-.10	.16	—		0.27	0.88	0.09
7 Venn	Ø	.15	.09	-.07	.37	-.07	—	0.14	0.64	0.05
Total	Ø	.47	.46	.15	.71	.48	.51			
Ø:	no variance				n = 22			$\bar{r}$ = 1.82		r_{tt} = .526

Table B.12. Correlation and reliability coefficients of PST on total presample

Problem number						$\bar{X}$	S
1 Equa	—					1.02	1.75
2 Bus	.23	—				2.17	1.38
3 Ant	.46	.28	—			1.76	0.73
4 Venn	.24	.22	.15	—		0.70	1.86
5 Fox	.36	-.05	.38	.32	—	1.68	1.58
Total	.73	.50	.60	.69	.64	7.32	4.68
n = 59		$\bar{r}$ = .516		r_{tt} = .809		Scale: 0 to 3	

Table B.13. Algebra grades predicted by means of stepwise regression

Variable	r	R^2	Variable	r	R^2
QUANT	.40	.16	NSBK	.26	.35
MARITH	.30	.19	VOCAB	.22	.35
SEX	.10	.21	MPFB	.09	.35
EPI-OP	.14	.23	MED	.01	.35
NSREAD	.35	.25	USES	.34	.36
VENN	.15	.26	BUS	.14	.36
MFIC	.22	.27	FOCC	-.06	.36
SDII-M	.12	.29	EXPR	.32	.36
SDII-G	-.06	.30	MOCC	-.04	.36
EPI-IW	.01	.31	SCON	.25	.36
SDII-VAL	.13	.32	SDII-MOT	-.01	.36
VOCAL	.05	.32	INSTR	.04	.36
SDII-R	.17	.33	SSREAD	.32	.36
SDII-EZ	.02	.33	FED	.03	.36
MNOSR	.25	.33	MGENFO	.17	.36
SDII-A	.04	.34	EPI-INC	.03	.36
EQUA	.24	.34	Variables not entered		
ANT	.02	.35	IQ	--	--
FOX	.15	.35	MVOC	--	--
			BUS	--	--
			LIT	--	--

Table B.14. Geometry grades predicted by means of stepwise regression

Variable	r	R^2	Variable	r	R^2
IQ	.57	.33	EPI-IW	.12	.56
EXPR	.53	.38	SDII-VAL	.11	.56
MARITH	.40	.43	LIT	.52	.56
NSREAD	.52	.45	VOCAB	.40	.57
EPI-OP	.13	.46	MOCC	-.02	.57
BUS	.20	.48	SDII-A	.02	.57
USES	.52	.49	EQUA	.33	.57
SDII-MOT	.06	.49	VENN	.11	.57
SCON	.43	.50	MED	.11	.58
QUANT	.56	.51	INSTR	.10	.58
SDII-EZ	.03	.51	MGENFO	.33	.58
SDII-M	.08	.52	NSBK	.40	.58
SDII-G	.14	.53	VOCAL	.16	.58
MFIG	.28	.53	FOX	.22	.58
SSREAD	.53	.53	MPFB	.15	.58
EPI-INC	.00	.54	SDII-R	.10	.58
SEX	.08	.55	FOCC	-.02	.58
FED	.14	.55	MVOC	.34	.58
MNOSR	.34	.56	ANT	.15	.58

Table B.15. Mathematics grades predicted by means of stepwise regression

Variable	r	R^2	Variable	r	R^2
QUANT	.54	.29	SDII-VAL	.13	.52
MARITH	.39	.34	SSREAD	.47	.52
USES	.48	.38	MOCC	-.04	.52
EPI-OP	.15	.40	SCON	.38	.53
NSREAD	.49	.42	IQ	.51	.53
SEX	.10	.44	VOCAB	.35	.53
MFIG	.28	.45	MPFB	.13	.53
SDII-M	.11	.46	BUS	.19	.53
SDII-EZ	.03	.47	VOCAL	.12	.53
VENN	.14	.48	LIT	.46	.53
MNOSR	.33	.49	INSTR	.08	.53
EPI-IW	.07	.50	SDII-R	.15	.53
SDII-A	.03	.50	SDII-G	.04	.53
EQUA	.32	.51	ANT	.10	.54
FED	.10	.51	FOCC	-.04	.54
SDII-MOT	.03	.51	MGENFO	.28	.54
FOX	.21	.51	Variables not entered		
EPI-INC	.02	.52	MED	--	--
NSBK	.37	.52	EXPR	--	--
			MVOC	--	--

Table B.16. Teacher Rating predicted by means of stepwise regression

Variable	r	R^2	Variable	r	R^2
QUANT	.50	.25	EPI-IW	.11	.49
MARITH	.45	.34	MVOC	.30	.49
LIT	.46	.38	SSREAD	.43	.49
BUS	.25	.40	MNOSR	.30	.49
EPI-OP	.11	.42	INSTR	.08	.49
EQUA	.38	.43	VOCAL	.13	.49
IQ	.50	.44	MGENFO	.29	.49
SEX	.47	.45	MED	.10	.49
VENN	.18	.46	MFIG	.17	.49
FED	.14	.46	SCON	.37	.49
ANT	.22	.47	EXPR	.46	.49
FOX	.24	.47	Variables not entered		
NSREAD	.44	.47	USES	--	--
NSBK	.30	.48	MOCC	--	--
SDII-A	.04	.48	SDII-G	--	--
MPFB	.13	.49	SDII-M	--	--
FOCC	-.05	.49	SDII-R	--	--
VOCAB	.34	.49	SDII-EZ	--	--
EPI-INC	.02	.49	SDII-MOT	--	--
			SDII-VAL	--	--

Table B.17. Semantic differential interest inventory[a]

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Mathematics	pleasant	-																					
	good	66	-																				
	active	59	47	-																			
	valuable	40	53	39	-																		
	strong	55	61	45	48	-																	
	love	70	54	50	35	54	-																
	fast	61	55	46	25	54	59	-															
	comfortable	75	62	53	43	53	68	67	-														
	nice	77	61	53	48	54	72	64	80	-													
	enjoyable	80	61	56	45	54	71	64	79	81	-												
	varied	27	24	29	23	16	19	19	24	32	27	-											
	soft	46	39	30	20	27	33	41	49	44	49	08	-										
	unafraid	39	28	29	07	26	35	38	41	44	37	16	37	-									
	light	39	26	27	07	19	29	34	36	34	34	19	44	31	-								
	secure	56	50	41	31	51	55	61	68	61	62	17	45	48	39	-							
Reading	pleasant	07	09	07	20	13	08	06	08	14	09	11	-07	-07	03	05	-						
	good	08	19	09	22	21	15	10	13	15	13	10	-05	00	04	07	71	-					
	active	04	05	12	08	08	09	03	04	10	08	16	-08	00	04	01	43	40	-				
	valuable	04	12	08	28	17	14	07	11	15	11	15	-13	-01	-03	10	61	65	33	-			
	strong	-05	-02	01	12	08	02	02	00	06	05	-0[illegible]	-07	-06	02	-02	67	59	45	48	-		
	love	08	13	11	17	15	20	13	14	14	13	05	-03	00	04	08	64	66	42	50	56	-	
	fast	01	-01	11	09	05	04	09	07	07	07	02	-09	06	05	09	56	50	35	46	63	54	-
	comfortable	04	11	12	18	14	12	19	11	14	13	04	-02	03	02	08	76	70	43	54	64	68	58
	nice	03	12	08	16	16	13	08	07	12	04	10	-09	-03	06	03	73	81	50	59	61	76	53
	enjoyable	06	16	06	28	17	13	05	11	16	12	12	-07	-04	-05	00	77	69	37	59	57	63	50
	varied	01	10	06	12	13	14	09	18	10	07	12	-07	03	00	13	41	47	67	31	31	38	34
	soft	-09	-06	-06	05	-04	02	-03	-04	-02	-03	04	01	-06	01	-02	50	47	31	35	51	52	49
	unafraid	-09	-02	-05	08	-00	03	-07	-06	02	-01	05	-11	02	-03	00	39	34	21	25	48	41	33
	light	-01	06	02	06	12	00	02	03	-01	-01	05	04	03	16	01	24	28	12	15	24	28	20
	secure	-01	04	08	21	12	14	02	09	09	07	14	-05	-03	02	07	64	59	37	50	62	61	59
Algebra	pleasant	71	55	53	29	51	62	62	66	67	72	25	40	37	33	60	05	07	-01	02	-08	07	03
	good	63	56	54	33	54	58	61	62	59	63	29	36	32	28	55	-05	03	-01	00	-16	04	-01
	active	48	43	61	30	39	42	43	44	41	46	21	22	26	23	41	01	05	03	05	-01	01	05
	valuable	51	53	40	50	47	42	39	45	44	47	21	23	12	14	39	08	11	01	13	-05	06	-04
	strong	47	48	41	25	53	45	56	51	48	49	17	29	22	20	51	01	05	-01	-00	-02	00	02
	love	64	49	53	33	48	68	55	61	65	67	25	32	36	29	54	00	05	04	05	-06	07	03
	fast	53	53	41	23	45	43	69	55	49	53	20	37	29	30	50	-09	-03	-06	-07	-08	-02	-00
	comfortable	65	55	48	30	49	56	60	65	67	67	24	41	40	36	62	02	05	-05	00	-11	03	02
	nice	72	57	52	35	52	65	62	68	72	70	31	42	37	33	58	02	04	-02	02	-11	03	-03
	enjoyable	70	57	51	35	54	62	62	64	66	72	26	41	33	32	55	01	01	03	-00	-12	04	-02
	varied	33	27	29	20	20	25	28	30	34	31	57	18	15	22	24	09	05	08	07	-05	03	00
	soft	44	32	30	11	32	35	40	43	40	42	17	49	37	39	42	-10	-13	-08	-18	-13	-08	-02
	unafraid	45	36	31	21	38	41	46	51	50	49	07	36	52	26	52	-08	-01	-04	-03	-13	01	01
	light	37	24	21	05	24	28	33	36	33	34	16	37	35	50	36	-13	-11	-07	-15	-17	-07	-13
	secure	58	42	40	24	44	51	56	60	57	60	19	35	40	27	64	-04	05	-03	-03	-04	04	04
Geometry	pleasant	57	41	41	22	36	53	49	59	61	57	14	38	40	28	47	04	03	01	08	01	08	01
	good	58	53	43	27	45	53	57	60	61	58	15	36	34	29	50	06	11	02	08	-05	09	-03
	active	39	40	46	26	31	40	38	37	39	39	09	22	22	26	39	11	12	07	15	01	14	05
	valuable	35	47	32	36	34	35	36	37	37	34	13	18	16	15	34	25	24	13	30	08	25	14
	strong	44	41	34	15	40	43	47	46	42	42	04	29	38	19	45	00	04	-02	01	03	03	06
	love	15	43	40	23	39	60	52	58	60	55	15	31	38	26	46	09	09	05	12	01	16	04
	fast	44	45	34	17	39	40	59	45	45	46	10	29	31	24	47	02	04	-07	05	03	02	12
	comfortable	56	45	40	22	38	49	54	59	60	60	11	40	44	34	53	04	07	03	06	-01	10	06
	nice	57	42	40	21	35	55	48	56	60	58	20	36	38	34	46	08	06	01	06	-01	09	02
	enjoyable	55	40	39	22	36	51	51	56	59	58	17	37	39	32	49	06	05	02	07	-02	06	04
	varied	22	22	22	09	25	24	26	23	22	20	35	16	10	13	25	08	05	17	05	-06	02	02
	soft	30	19	16	03	18	27	30	37	31	36	03	43	27	31	30	-02	-08	-02	-08	-01	03	-04
	unafraid	36	31	20	12	24	26	36	41	42	37	19	24	50	29	41	04	03	05	05	-03	07	00

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
-																												
71	-																											
43	40	-																										
61	65	33	-																									
67	59	45	48	-																								
64	66	42	50	56	-																							
56	50	35	46	63	54	-																						
76	70	43	54	64	68	58	-																					
73	81	50	59	61	76	53	71	-																				
77	69	37	59	57	63	50	69	67	-																			
41	47	67	31	31	38	34	43	47	40	-																		
50	47	31	35	51	52	49	52	49	48	35	-																	
39	34	21	25	48	41	33	42	32	33	24	43	-																
24	28	12	15	24	28	20	30	29	28	20	38	26	-															
54	59	37	50	62	61	59	64	62	61	49	56	51	30	-														
05	07	-01	02	-08	07	03	02	01	03	08	-10	-11	-03	-02	-													
05	03	-01	00	-16	04	-01	-04	-02	-02	14	-13	-13	-03	01	08	-												
01	05	03	05	-01	01	05	02	05	-06	06	-12	-09	-08	01	63	56	-											
08	11	01	13	-05	06	-04	04	02	09	12	-06	-07	-05	08	58	61	46	-										
01	05	-01	-00	-02	00	02	-03	01	-02	16	-05	-12	00	04	65	62	48	45	-									
00	05	04	05	-06	07	03	-02	-02	-00	06	-06	-05	-09	01	83	75	60	63	60	-								
09	-03	-06	-07	-08	-02	-00	-08	-06	-08	04	-19	-17	-08	-11	68	67	54	52	61	61	-							
02	05	-05	00	-11	03	02	01	-00	00	07	-09	-12	-06	-06	85	74	54	52	62	75	66	-						
02	04	-02	02	-11	03	-03	-01	01	-01	08	-11	-11	-03	-01	88	78	60	61	67	82	70	82	-					
1	01	03	-00	-12	04	-02	-02	-03	-01	05	-09	-11	00	01	88	80	60	60	65	83	68	77	85	-				
9	05	08	07	-05	03	00	05	09	06	03	-07	01	03	10	36	35	29	37	27	34	29	35	41	39	-			
0	-13	-08	-18	-13	-08	-02	-09	-15	-13	-04	-05	-13	01	-11	57	50	31	33	39	53	50	59	55	57	23	-		
8	-01	-04	-03	-13	01	01	-07	-09	-03	07	-00	01	-04	-04	58	54	31	38	42	59	50	61	56	56	18	44	-	
3	-11	-07	-15	-17	-07	-13	-17	-08	-15	-06	-13	-19	01	-16	46	39	26	20	32	44	42	47	45	43	25	62	39	-
4	05	-03	-03	-04	04	04	-03	-06	-06	09	-05	-04	-03	03	76	72	54	53	61	75	67	75	75	72	29	54	59	41
4	03	01	08	01	08	01	08	03	01	07	-09	-06	04	-03	57	45	38	30	40	51	43	54	59	51	18	31	39	22
06	11	02	08	-05	09	-03	07	07	03	10	-13	-09	04	-02	62	56	43	42	48	55	53	58	65	59	19	36	42	25
11	12	07	15	01	14	05	10	13	03	10	-08	-05	01	-02	47	35	49	30	43	42	37	43	47	42	11	20	26	14
25	24	13	30	08	25	14	25	19	22	24	02	03	04	15	42	41	36	58	37	41	38	37	44	39	20	18	27	10
00	04	-02	01	03	03	06	09	01	-02	06	-13	-07	03	-06	44	33	31	19	47	36	47	43	44	39	09	29	30	21
09	09	05	12	01	16	04	13	06	05	11	-11	-01	-02	-02	57	46	36	32	38	59	46	54	60	52	19	32	39	26
02	04	-07	05	03	02	12	07	02	00	11	-07	-02	-06	-03	53	45	38	28	45	43	62	53	53	46	14	35	35	22
04	07	03	06	-01	10	06	08	06	-01	06	-10	-04	04	-03	61	48	36	27	41	49	51	59	60	54	16	39	44	31
08	06	01	06	-01	09	02	08	07	01	08	-07	-06	01	-02	59	48	39	33	39	54	44	55	62	51	20	37	37	27
06	05	02	07	-02	06	04	10	02	03	07	-10	-06	02	-05	61	52	39	35	36	54	48	60	63	55	19	35	41	23
08	05	17	05	-06	02	02	02	06	02	11	-01	-03	08	02	27	27	22	19	23	26	24	22	29	27	40	17	14	10
02	-08	-02	-08	-01	03	-04	-02	-04	-07	02	-02	-05	09	-05	29	19	12	10	16	26	27	29	30	25	06	44	27	37
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
-																							
66	-																						
70	82	-																					
68	77	85	-																				
29	35	41	39	-																			
50	59	55	57	23	-																		
50	61	56	56	18	44	-																	
42	47	45	43	25	62	39	-																
67	75	75	72	29	54	59	41	-															
43	54	59	51	18	31	39	22	45	-														
53	58	65	59	19	36	42	25	53	79	-													
37	43	47	42	11	20	26	14	35	59	59	-												
38	37	44	39	20	18	27	10	35	47	57	52	-											
47	43	44	39	09	29	30	21	39	63	69	50	42	-										
46	54	60	52	19	32	39	26	46	79	79	55	54	66	-									
62	53	53	46	14	35	35	22	46	67	70	49	46	68	66	-								
51	59	60	54	16	39	44	31	48	80	79	55	48	72	77	73	-							
44	55	62	51	20	37	37	27	45	84	77	62	53	62	80	66	80	-						
48	60	63	55	19	35	41	23	50	82	77	53	53	59	79	69	79	77	-					
24	22	29	27	40	17	14	10	21	29	31	27	28	16	27	23	21	26	26	-				
27	29	30	25	06	44	27	37	23	51	51	25	22	43	52	46	56	52	50	14	-			

Mathematics	pleasant	-																							
	good	66	-																						
	active	59	47	-																					
	valuable	40	53	39	-																				
	strong	55	61	45	48	-																			
	love	70	54	50	35	54	-																		
	fast	61	55	46	25	54	59	-																	
	comfortable	75	62	53	43	53	68	67	-																
	nice	77	61	53	48	54	72	64	80	-															
	enjoyable	80	61	56	45	54	71	64	79	81	-														
	varied	27	24	29	23	16	19	19	24	32	27	-													
	soft	46	39	30	20	27	33	41	49	44	49	08	-												
	unafraid	39	28	29	07	26	35	38	41	44	37	16	37	-											
	light	39	26	27	07	19	29	34	36	34	34	19	44	31	-										
	secure	56	50	41	31	51	55	61	68	61	62	17	45	48	39	-									
Reading	pleasant	07	09	07	20	13	08	06	08	14	09	11	-07	-07	03	05	-								
	good	08	19	09	22	21	15	10	13	15	13	10	-05	00	04	07	71	-							
	active	04	05	12	08	08	09	03	04	10	08	16	-08	00	04	01	43	40	-						
	valuable	04	12	08	28	17	14	07	11	15	11	15	-13	-01	-03	10	61	65	33	-					
	strong	-05	-02	01	12	08	02	02	00	06	05	-02	-07	-06	02	-02	67	59	45	48	-				
	love	08	13	11	17	15	20	13	14	14	13	05	-03	00	04	08	64	66	42	50	56	-			
	fast	01	-01	11	09	05	04	09	07	07	07	02	-09	06	05	09	56	50	35	46	63	54	-		
	comfortable	04	11	12	18	14	12	19	11	14	13	04	-02	03	02	08	76	70	43	54	64	68	58	-	
	nice	03	12	08	16	16	13	08	07	12	04	10	-09	-03	06	03	73	81	50	59	61	76	53	71	-
	enjoyable	06	16	06	28	17	13	05	11	16	12	12	-07	-04	-05	00	77	69	37	59	57	63	50	69	67
	varied	01	10	06	12	13	14	09	18	10	07	12	-07	03	00	13	41	47	67	31	31	38	34	43	47
	soft	-09	-06	-06	05	-04	02	-03	-04	-02	-03	04	01	-06	01	-02	50	47	31	35	51	52	49	52	49
	unafraid	-09	-02	-05	08	-00	03	-07	-06	02	-01	05	-11	02	-03	00	39	34	21	25	48	41	33	42	32
	light	-01	06	02	06	12	00	02	03	-01	-01	05	04	03	16	01	24	28	12	15	24	28	20	30	29
	secure	-01	04	08	21	12	14	02	09	09	07	14	-05	-03	02	07	64	59	37	50	62	61	59	64	62
Algebra	pleasant	71	55	53	29	51	62	62	66	67	72	25	40	37	33	60	05	07	-01	02	-08	07	03	02	01
	good	63	56	54	33	54	58	61	62	59	63	29	26	32	28	55	-05	03	-01	00	-16	04	-01	-04	-02
	active	48	43	61	30	39	42	43	44	41	46	21	22	26	23	41	01	05	03	05	-01	01	05	02	05
	valuable	51	53	40	50	47	42	39	45	44	47	21	23	12	14	39	08	11	01	13	-05	06	-04	04	02
	strong	47	48	41	25	53	45	56	51	48	49	17	29	22	20	51	01	05	-01	-00	-02	00	02	-03	01
	love	64	49	53	33	48	68	55	61	65	67	25	32	36	29	54	00	05	04	05	-06	07	03	-02	-02
	fast	53	53	41	23	45	43	69	55	49	53	20	37	29	30	50	-09	-03	-06	-07	-08	-02	-00	-08	-06
	comfortable	65	55	48	30	49	56	60	65	67	67	24	41	40	36	62	02	05	-05	00	-11	03	02	01	-00
	nice	72	57	52	35	52	65	62	68	72	70	31	42	37	33	58	02	04	-02	02	-11	03	-03	-01	01
	enjoyable	70	57	51	35	54	62	62	64	66	72	26	41	33	32	55	01	01	03	-00	-12	04	-02	-02	-03
	varied	33	27	29	20	20	25	28	30	34	31	57	18	15	22	24	09	05	08	07	-05	03	00	05	09
	soft	44	32	30	11	32	35	40	43	40	42	17	49	37	39	42	-10	-13	-08	-18	-13	-08	-02	-09	-15
	unafraid	45	36	31	21	38	41	46	51	50	49	07	36	52	26	52	-08	-01	-04	-03	-13	01	01	-07	-09
	light	37	24	21	05	24	28	33	36	33	34	16	37	35	50	36	-13	-11	-07	-15	-17	-07	-13	-17	-08
	secure	58	42	40	24	44	51	56	60	57	60	19	35	40	27	64	-04	05	-03	-03	-04	04	04	-03	-06
Geometry	pleasant	57	41	41	22	36	53	49	59	61	57	14	38	40	28	47	04	03	01	08	01	08	01	08	03
	good	58	53	43	27	45	53	57	60	61	58	15	36	34	29	50	06	11	02	08	-05	09	-03	07	07
	active	39	40	46	26	31	40	38	37	39	39	09	22	22	26	39	11	12	07	15	01	14	05	10	13
	valuable	35	47	32	36	34	35	36	37	37	34	13	18	16	15	34	25	24	13	30	08	25	14	25	19
	strong	44	41	34	15	40	43	47	46	42	42	04	29	38	19	45	00	04	-02	01	03	03	06	09	01
	love	15	43	40	23	39	60	52	58	60	55	15	31	36	26	46	09	09	05	12	01	16	04	13	06
	fast	44	45	34	17	39	40	59	45	45	46	10	29	31	24	47	02	04	-07	05	03	02	12	07	02
	comfortable	56	45	40	22	38	49	54	59	60	60	11	40	44	34	53	04	07	03	06	-01	10	06	08	06
	nice	57	42	40	21	35	55	48	56	60	58	20	36	38	34	46	08	06	01	06	-01	09	02	08	07
	enjoyable	55	40	39	22	36	51	51	56	59	58	17	37	39	32	49	06	05	02	07	-02	06	04	10	02
	varied	22	22	22	09	25	24	26	23	22	20	35	16	10	13	25	08	05	17	05	-06	02	02	02	06
	soft	30	19	16	03	18	27	30	37	31	36	03	43	27	31	30	-02	-08	-02	-08	-01	03	-04	-02	-04
	unafraid	36	31	20	12	24	26	36	41	42	37	19	24	50	29	41	04	03	05	05	-03	07	00	05	01
	light	28	18	12	09	15	21	19	30	25	27	08	25	20	36	24	03	01	-06	-06	02	05	04	02	05
	secure	51	41	34	20	39	49	52	55	56	54	08	32	47	31	57	06	06	02	09	01	05	04	02	05

[a]Decimals omitted.

-																												
71	-																											
43	40	-																										
61	65	33	-																									
67	59	45	48	-																								
64	66	42	50	56	-																							
56	50	35	46	63	54	-																						
76	70	43	54	64	68	58	-																					
73	81	50	59	61	76	53	71	-																				
77	69	37	59	57	63	50	69	67	-																			
41	47	67	31	31	38	34	43	47	40	-																		
50	47	31	35	51	52	49	52	49	48	35	-																	
39	34	21	25	48	41	33	42	32	33	24	43	-																
24	28	12	15	24	28	20	30	29	28	20	38	26	-															
64	59	37	50	62	61	59	64	62	61	49	56	51	30	-														
05	07	-01	02	-08	07	03	02	01	03	08	-10	-11	-03	-02	-													
-05	03	-01	00	-16	04	-01	-04	-02	-02	14	-13	-13	-03	01	08	-												
01	05	03	05	-01	01	05	02	05	-06	06	-12	-09	-08	01	63	56	-											
08	11	01	13	-05	05	-04	04	02	09	12	-06	-07	-05	08	58	61	46	-										
01	05	-01	-00	-02	00	02	-03	01	-02	16	-05	-12	00	04	65	62	48	45	-									
00	05	04	05	-06	07	03	-02	-02	-00	06	-06	-05	-09	01	83	75	60	63	60	-								
-09	-03	-06	-07	-08	-02	-00	-08	-06	-08	04	-19	-17	-08	-11	68	67	54	52	61	61	-							
02	05	-05	00	-11	03	02	01	-00	00	07	-09	-12	-06	-06	85	74	54	52	62	75	66	-						
02	04	-02	02	-11	03	-03	-01	01	-01	08	-11	-11	-03	-01	88	78	60	61	67	82	70	82	-					
01	01	03	-00	-12	04	-02	-02	-03	-01	05	-09	-11	00	01	88	80	60	60	65	83	68	77	85	-				
09	05	08	07	-05	03	00	05	09	06	03	-07	01	03	10	36	35	29	37	27	34	29	35	41	39	-			
-10	-13	-08	-18	-13	-08	-02	-09	-15	-13	-04	-05	-13	01	-11	57	50	31	33	39	53	50	59	55	57	23	-		
-08	-01	-04	-03	-13	01	01	-07	-09	-03	07	-00	01	-04	-04	58	54	31	38	42	59	50	61	56	56	18	44	-	
-13	-11	-07	-15	-17	-07	-13	-17	-08	-15	-06	-13	-19	01	-16	46	39	26	20	32	44	42	47	45	43	25	62	39	
-04	05	-03	-03	-04	04	04	-03	-06	-06	09	-05	-04	-03	03	76	72	54	53	61	75	67	75	75	72	29	54	59	[illegible]
04	03	01	08	01	08	01	08	03	01	07	-09	-06	04	-03	57	45	38	30	40	51	43	54	59	51	18	31	39	[illegible]
06	11	02	08	-05	09	-03	07	07	03	10	-13	-09	04	-02	62	56	43	42	48	55	53	58	65	59	19	36	42	[illegible]
11	12	07	15	01	14	05	10	13	03	10	-08	-05	01	-02	47	35	49	30	43	42	37	43	47	42	11	20	26	[illegible]
25	24	13	30	08	25	14	25	19	22	24	02	03	04	15	42	41	36	58	37	41	38	37	44	39	20	18	27	[illegible]
00	04	-02	01	03	03	06	09	01	-02	06	-13	-07	03	-06	44	33	31	19	47	36	47	43	44	39	09	29	30	[illegible]
09	09	05	12	01	16	04	13	06	05	11	-11	-01	-02	-02	57	46	36	32	38	59	46	54	60	52	19	32	39	[illegible]
02	04	-07	05	03	02	12	07	02	00	11	-07	-02	-06	-03	53	45	38	28	45	43	62	53	53	46	14	35	35	[illegible]
04	07	03	06	-01	10	06	08	06	-01	06	-10	-04	04	-03	61	48	36	27	41	49	51	59	60	54	16	39	44	[illegible]
08	06	01	06	-01	09	02	08	07	01	08	-07	-06	01	-02	59	48	39	33	39	54	44	55	62	51	20	37	37	[illegible]
06	05	02	07	-02	06	04	10	02	03	07	-10	-06	02	-05	61	52	39	35	36	54	48	60	63	55	19	35	41	[illegible]
08	05	17	05	-06	02	02	02	06	02	11	-01	-03	08	02	27	27	22	19	23	26	24	22	29	27	40	17	14	[illegible]
-02	-08	-02	-08	-01	03	-04	-02	-04	-07	02	-02	-05	09	-05	29	19	12	10	16	26	27	29	30	25	06	44	27	[illegible]
04	03	05	05	-03	07	00	05	01	01	05	-10	04	09	-07	35	28	21	17	30	31	35	41	39	32	16	25	47	[illegible]
03	01	-06	-06	02	05	04	02	05	01	04	-02	-03	07	01	27	21	17	15	11	22	26	26	24	23	09	37	20	[illegible]
06	06	02	09	01	05	04	02	05	01	04	-02	-03	07	01	54	46	33	26	44	48	47	53	56	48	17	32	46	[illegible]

-																							
66	-																						
70	82	-																					
68	77	85	-																				
29	35	41	39	-																			
50	59	55	57	23	-																		
50	61	56	56	18	44	-																	
42	47	45	43	25	62	39	-																
67	75	75	72	29	54	59	41	-															
43	54	59	51	18	31	39	22	45	-														
53	58	65	59	19	36	42	25	53	79	-													
37	43	47	42	11	20	26	14	35	59	59	-												
38	37	44	39	20	18	27	10	35	47	57	52	-											
47	43	44	39	09	29	30	21	39	63	69	50	42	-										
46	54	60	52	19	32	39	26	46	79	79	55	54	66	-									
62	53	53	46	14	35	35	22	46	67	70	49	46	68	66	-								
51	59	60	54	16	39	44	31	48	80	79	55	48	72	77	73	-							
44	55	62	51	20	37	37	27	45	84	77	62	53	62	80	66	80	-						
48	60	63	55	19	35	41	23	50	82	77	53	53	59	79	69	79	77	-					
24	22	29	27	40	17	14	10	21	29	31	27	28	16	27	23	21	26	26	-				
27	29	30	25	06	44	27	37	23	51	51	25	22	43	52	46	56	52	50	14	-			
35	41	39	32	16	25	47	29	37	59	55	43	31	54	55	50	65	58	55	13	40	-		
26	26	24	23	09	37	20	44	24	33	41	18	17	34	39	38	43	38	39	13	60	33	-	
47	53	56	48	17	32	46	26	50	73	74	53	46	70	73	72	79	70	75	24	53	67	40	-

Table B.18. Correlation matrix of dependent and independent variables[a]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1 AL1	-																		
2 AL2	79	-																	
3 GE1	53	57	-																
4 GE2	53	57	82	-															
5 TR	48	55	66	67	-														
6 QUANT	38	37	54	53	50	-													
7 MVOC	20	18	35	31	30	42	-												
8 MGENFO	12	20	31	32	29	38	49	-											
9 MARITH	27	30	34	42	45	35	11	20	-										
10 MNOSR	23	24	31	34	30	35	18	22	34	-									
11 MFIG	23	18	25	28	17	24	23	21	20	28	-								
12 MPFB	10	07	15	13	13	28	16	14	20	16	20	-							
13 SEX	09	11	11	06	07	-14	04	-11	-08	-21	-10	-21	-						
14 IQ	34	32	57	53	50	69	53	44	35	38	35	29	-11	-					
15 SCON	22	25	46	37	37	62	56	49	17	23	22	14	-15	66	-				
16 NSPK	23	27	42	34	30	59	49	39	13	21	22	18	-25	56	76	-			
17 EXPR	30	31	54	47	56	61	47	31	25	21	17	09	25	59	60	47	-		
18 SSREAD	29	31	52	48	43	59	54	41	23	22	15	10	01	61	75	63	67	-	
19 NSREAD	31	36	52	48	44	67	52	40	16	23	20	21	-01	61	74	73	66	80	-
20 LIT	26	31	52	48	46	61	52	38	19	21	15	06	12	61	74	63	80	75	73
21 VOCAB	17	23	43	35	34	55	63	47	14	15	19	11	-07	64	80	70	63	71	69
22 USES	32	32	54	47	41	56	51	41	22	22	19	10	14	60	68	57	69	69	66
23 MED	01	01	10	11	10	07	16	07	06	08	-04	02	-05	14	17	16	06	16	16
24 FED	02	03	12	16	14	09	14	10	-04	06	-07	07	01	18	12	15	10	14	17
25 MOCC	-04	-04	-03	-02	04	05	07	-02	-03	-06	06	-02	-05	06	01	-05	01	03	00
26 FOCC	-06	-06	-01	-02	-05	-07	-05	-03	04	07	-05	00	03	-04	-07	01	-02	-07	-03
27 INSTR	04	03	10	09	08	08	04	06	06	09	10	06	06	11	07	04	10	06	10
28 VOCAL	06	04	17	14	13	19	08	13	15	06	08	07	15	12	10	04	19	14	11
29 EQUA	22	24	30	34	38	35	21	23	27	27	17	16	-04	34	31	25	29	31	29
30 BUS	12	14	19	19	25	16	17	16	06	20	08	10	07	19	15	19	13	18	16
31 ANT	-03	06	14	14	22	17	16	24	15	18	10	08	-07	25	23	27	19	22	19
32 VENN	16	13	09	12	18	07	08	10	13	04	-01	10	02	11	03	06	03	07	04
33 FOX	13	15	23	19	24	22	16	15	09	13	22	16	02	31	22	19	16	19	19
34 EPI-IW	-03	04	13	10	11	23	27	31	05	15	48	12	-20	26	35	38	16	25	26
35 EPI-OP	11	15	11	13	11	-03	-11	-08	04	00	03	13	09	06	10	-16	02	-09	-08
36 EPI-INC	05	00	03	-02	02	14	13	08	-03	-05	01	05	01	17	20	16	11	16	12
37 SDII-A	08	00	02	01	04	-07	-05	01	04	-08	-00	-03	02	02	-04	-04	04	03	-02
38 SDII-G	-07	-06	13	14	08	11	05	06	02	-01	03	-00	-06	10	13	15	10	11	15
39 SDII-M	11	12	10	05	09	02	04	00	-07	10	-10	07	16	05	-01	00	13	02	05
40 SDII-R	16	17	09	11	08	12	03	11	08	02	06	03	09	14	12	18	12	13	14
41 SDII-EZ	03	01	06	-00	11	-11	08	02	-08	13	03	14	-01	11	14	12	19	13	17
42 SDII-MOT	01	-03	06	06	-04	03	03	07	07	11	11	06	-09	04	07	01	-06	03	02
43 SDII-VAL	15	09	12	09	03	-00	13	04	03	-03	05	00	11	03	04	07	07	05	07

[a]Decimals omitted.

dent variables[a]

12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
-																								
-21	-																							
29	-11	-																						
14	-15	66	-																					
18	-25	56	76	-																				
09	25	59	60	47	-																			
10	01	61	75	63	67	-																		
21	-01	61	74	73	66	80	-																	
06	12	61	74	63	80	75	73	-																
11	-07	64	80	70	63	71	69	77	-															
10	14	60	68	57	69	69	66	70	67	-														
02	-05	14	17	16	06	16	16	08	14	14	-													
07	01	18	12	15	10	14	17	10	15	09	40	-												
-02	-05	06	01	-05	01	03	00	-01	-02	-01	25	05	-											
00	03	-04	-07	01	-02	-07	-03	-02	-05	-02	10	22	09	-										
06	06	11	07	04	10	06	10	13	07	10	07	14	04	07	-									
07	15	12	10	04	19	14	11	21	14	18	-01	-04	05	04	25	-								
16	-04	34	31	25	29	31	29	32	24	31	03	01	09	-11	-01	11	-							
10	07	19	15	19	13	18	16	19	14	18	-02	17	03	-03	10	01	20	-						
08	-07	25	23	27	19	22	19	23	23	17	02	05	-04	03	03	02	16	21	-					
10	02	11	03	06	03	07	04	02	02	02	03	03	04	-02	04	03	16	12	10	-				
16	02	31	22	19	16	19	19	24	19	20	06	-03	02	-10	-02	19	18	13	15	08	-			
12	-20	26	35	38	16	25	26	21	32	23	07	06	-03	02	05	-01	20	04	18	06	12	-		
13	09	06	10	-16	02	-09	-08	-04	-13	-06	-07	-04	01	01	01	03	00	-16	-25	-04	-01	-03	-	
05	01	17	20	16	11	16	12	15	19	11	03	07	-07	-10	-05	12	-02	00	04	02	07	-18	-06	-
-03	02	02	-04	-04	04	03	-02	-01	-01	03	06	-03	12	03	02	-03	-08	-01	-18	-02	01	09	10	-12
-00	-06	10	13	15	10	11	15	13	12	05	00	03	12	-01	04	-01	03	11	09	02	08	-03	-03	12
07	16	05	-01	00	13	02	05	05	02	07	-04	03	-02	-05	-01	10	06	03	11	01	-00	-00	-06	03
03	09	14	12	18	12	13	14	16	14	16	03	08	-04	03	15	14	03	01	01	03	01	05	-08	-03
14	-01	11	14	12	19	13	17	14	09	10	01	01	18	-01	-02	11	09	02	08	04	-07	01	-05	09
06	-09	04	07	01	-06	03	02	-06	-01	-04	-00	01	-00	-04	06	-07	-00	-00	-04	03	-04	00	-04	03
00	11	03	04	07	07	05	07	04	02	11	08	07	02	00	15	11	01	04	-11	01	04	08	05	02

24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
-																			
05	-																		
22	09	-																	
14	04	07	-																
-04	05	04	25	-															
01	09	-11	-01	11	-														
17	03	-03	10	01	20	-													
05	-04	03	03	02	16	21	-												
03	04	-02	04	03	16	12	10	-											
-03	02	-10	-02	19	18	13	15	08	-										
06	-03	02	05	-01	20	04	18	06	12	-									
-04	01	01	01	03	00	-16	-25	-04	-01	-03	-								
07	-07	-10	-05	12	-02	00	04	02	07	-18	-06	-							
-03	12	03	02	-03	-08	-01	-18	-02	01	09	10	-12	-						
03	12	-01	04	-01	03	11	09	02	08	-03	-03	12	-15	-					
03	-02	-05	-01	10	06	03	11	01	-00	-00	-06	03	-32	-02	-				
08	-04	03	15	14	03	01	01	03	01	05	-08	-03	25	-10	09	-			
01	18	-01	-02	11	09	02	08	04	-07	01	-05	09	-04	23	40	10	-		
01	-00	-04	06	-07	-00	-00	-04	03	-04	00	-04	03	-10	11	-05	-14	03	-	
07	02	00	15	11	01	04	-11	01	04	08	05	02	27	06	00	20	-15	-22	-